Relativity, Inequality, and Optimal Nonlinear Income Taxation in an Open Economy*

Darong Dai† Wenzheng Gao‡ Guoqiang Tian§

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Abstract

Recent evidence suggests that globalization has not just reduced the barriers to international labor mobility but also induced more cross-country social comparisons. This paper extends the analysis of Kanbur and Tuomala (2013) (K&T) to an open economy with tax-driven migrations and cross-country consumption comparisons. We derive an optimal tax formula which subsumes existing formulas obtained under maximin social objective and additively separable utility, and we qualitatively characterize this formula over a continuum of income distribution. We establish thresholds of the elasticity and level of migration to identify when relativity and inequality are complementary (or substitutive) in shaping the tax rates of top-income workers, generalizing the prediction of K&T obtained under a zero elasticity and level of migration. Under both Nash and Stackelberg tax competition: if the migration probability of top-income workers is around 50%, numerical calculation using realistic parameter values shows that the country with labor inflow (resp. outflow) imposes over 10% lower (resp. higher) marginal tax rates than suggested by K&T.

Keywords: Relative consumption; Income inequality; International labor mobility; Maximin; Optimal income taxation; Tax competition.

JEL classification codes: D63; H21; H23; J61.

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†E-mail: dai496@tamu.edu. Department of Economics, Texas A&M University, College Station 77843, USA.
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‡E-mail: orcgwz@tamu.edu. Department of Economics, Texas A&M University, College Station 77843, USA.

§E-mail: gtian@tamu.edu. Department of Economics, Texas A&M University, College Station 77843, USA.

1 Introduction

It was recognized by economists (e.g., Veblen, 1899; Duesenberry, 1949; Alpizar et al., 2005; Luttmer, 2005; Andersson, 2008; Clark et al., 2008) that the well-being of economic agents depends on relative consumption in addition to absolute consumption, no matter they are motivated by jealousy or altruism. As such, taxing consumption externalities seems to be welfare enhancing as any other Pigouvian tax. In this paper we focus on the tool of labor income tax and design optimal nonlinear tax schedules to deal with income inequality and consumption externality in an open economy with international labor mobility. In particular, for top-income workers, we ask how income inequality and consumption relativity together determine their tax rates. This question was addressed by Kanbur and Tuomala (2013) (K&T hereafter) in a closed economy with a single government. Our goal is to answer this question in an open economy with two governments who compete for taxpayers.

The policy relevance of tax-driven migrations has been empirically supported by Kleven et al. (2013), Kleven et al. (2014) and Akcigit et al. (2016) who estimate large elasticities of migration with respect to tax rate for highly skilled workers. Using survey-data for countries in Western Europe from the early 1970s to 2002, Becchetti et al. (2013) find that the contribution of cross-country comparisons to well-being increased over the study period. Piketty (2014) even argues that cross-country social comparisons seem to constitute an important part of the motivation behind Thatcher's and Reagan's drastic income tax reductions in the early 1980s.

We therefore investigate in what respects strategic tax competition and cross-country social comparisons change the answers suggested by K&T. For this purpose, we focus on income tax schedules that competing governments find optimal to implement in two types of non-cooperative equilibrium: Nash and Stackelberg. We start with the Nash solution, where each country takes the strategy of the opponent country as given. Each government fully internalizes consumption externalities affecting workers within its own country, but completely ignore the externalities affecting the opponent country.

As argued by Aronsson and Johansson-Stenman (2015), Nash competition is not necessarily the most realistic one since the ability to commit to pubic policy may differ among countries. We thus analyze a Stackelberg equilibrium where one country acts as the leader with the opponent country acting as the follower. As is canonical, the leader shall recognize the behavioral responses of the follower and hence take into account the externalities it causes to the follower country. This, accordingly, implies that optimal tax schedules in these two types of equilibrium are in general different for the leader country.

In each country, workers differ in both skills and migration costs, which are assumed to be private information. We hence follow the mechanism design approach. Taking as given income taxes, workers make individual decisions along two margins. The allocation of one-unit time between work and leisure on the intensive margin, and the location choice on the extensive margin. To make the analysis more transparent, we restrict attention to the most redistributive social objective maximin[1] in the spirit of Rawls (1971). As a result, after taking into account individual responses, each government designs incentive-compatible allocations such that the utility of the worst-off is maximized and the public-sector budget constraint is satisfied. Throughout, taxes can only be conditioned on income and are levied according to the residence principle.

We characterize the best response of each government and obtain a formula determining optimal marginal tax rates. The optimal tax formula obtained by Oswald (1983) and K&T for a closed economy is augmented by a migration effect which affects both the Pigouvian-tax term and the Mirrleesian-tax term in the tax formula, leading to a much more complicated formula for qualitative analysis. In addition, following Lehmann et al. (2014), we also derive an optimal tax formula under the useful benchmark called the Tiebout-best, in which workers' skills are assumed to be common knowledge while migration costs remain private information. By eliminating the standard incentive-compatibility constraints, the maximization problem of tax design becomes much more simple. In fact, we can explicitly solve for both the Tiebout-best tax liability and the Tiebout-best marginal tax rate.

The major theoretical predictions can be summarized as follows. First, for the case where workers exhibit jealousy-type relativity: (i) if the skill distribution is bounded and the tax liability is smaller than the Tiebout-best tax liability, then the second-best marginal tax rates in both Nash and Stackelberg equilibrium are positive over the entire income range; (ii) we identify mild conditions such that the Mirrleesian-tax term in the optimal tax formula is negative over entire income range but the endpoints; (iii) for the leader country under Stackelberg competition, marginal tax rate is always higher than that under Nash competition. Second, for the case where workers exhibit altruism-type relativity, we identify mild conditions such that the Mirrleesian-tax term in the optimal tax formula is positive over entire income range but the endpoints.

To analyze how relativity and inequality together shape the marginal tax rate imposed on top-income workers, we obtain a closed-from formula of the optimal asymptotic marginal tax rate. By using realistic parameter values from empirical studies, we numerically calculate these

[1]As is demonstrated by Boadway and Jacquet (2008), focusing on the maximin objective significantly simplifies the analytical analysis of the optimal income tax structure.

tax rates under both types of equilibrium and compare them to those calculated using the K&T-formula. In both Nash and Stackelberg equilibrium, the country with large labor inflow imposes a much smaller marginal tax rate than suggested by K&T, while the country with large labor outflow imposes a much higher tax rate than suggested by K&T. Also, the asymptotic marginal tax rate in the Stackelberg equilibrium is higher than that in the Nash equilibrium. As a policy implication for open economies, these results reveal that normative public policy recommendations on redistributive income taxes must take between-country tax competition, tax-driven migrations and relative consumption concerns seriously, otherwise workers are likely to face welfare losses.

Our study is also related the literature studying optimal nonlinear income taxation in an open economy, such as Mirrlees (1982), Simula and Trannoy (2010), Bierbrauer et al. (2013), Lehmann et al. (2014), Aronsson and Johansson-Stenman (2015), and Blumkin et al. (2015). The major difference between these studies and our paper is that we focus on examining how the interplay of relativity and inequality determines the optimal nonlinear income tax schedule, which is however ignored by these studies. Except Aronsson and Johansson-Stenman (2015), the other literatures just completely ignore the effect of relative consumption concern placed on the design of Mirrlees income taxes. As numerically illustrated in Section 5, relative consumption concern does result in quantitatively significant effects on the optimal marginal tax rates and hence should not be ignored. Though Aronsson and Johansson-Stenman (2015) also consider both tax competition and relative consumption concerns, they assume away the possibility of labor mobility between countries, whereas we have shown that migrations can shape the tax-competition effect and hence equilibrium tax rates in an important way. In sum, our study extends the literature and proves the relevance of taking into account tax-driven migrations and relative consumption concerns in designing optimal nonlinear income taxes.

The remainder of the paper is organized as follows. Section 2 sets up the model. Section 3 derives the optimal tax formula in Nash equilibrium and establishes some qualitative properties. Section 4 derives the optimal tax formula in Stackelberg equilibrium and establishes some qualitative properties. Section 5 provides some numerical examples regarding the optimal asymptotic marginal tax rates and compares our results with those calculated using K&T-formula. Section 6 concludes. All formal proofs are relegated to Appendix.

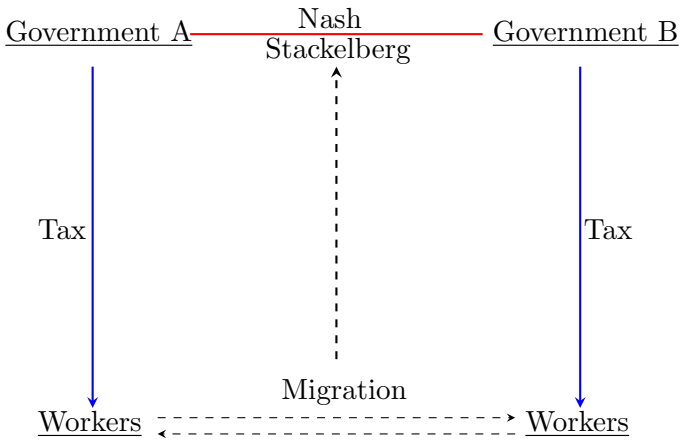

Figure 1: Agents and Relationships.

2 The Model

We consider an economy consisting of two countries, indexed by $i \in \{A, B\}$. The measure of workers in country i is normalized to 1, while that of the opponent country $-i$ is denoted by n_{-i}, for $0 < n_{-i} \leq 1$. Each worker is characterized by three characteristics: her native country $i \in \{A, B\}$, her skill $w \in [\underline{w}, \overline{w}]$ with $0 < \underline{w} < \overline{w} \leq \infty$, and the migration cost $m \in \mathbb{R}^+$ she supports if she decides to live abroad. If a worker faces an infinitely large migration cost, then she is immobile. Following Lehmann et al. (2014), we do not make any restriction on the correlation between skills and migration costs.

The skill density function in country i, $f_i(w) = F_i'(w) > 0$, is assumed to be differentiable for all $w \in [\underline{w}, \overline{w}]$ and is single-peaked, with a mode at w_m. For each skill w, $g_i(m|w)$ denotes the conditional density of the migration cost and $G_i(m|w) = \int_0^m g_i(x|w)dx$ the conditional cumulative distribution function. The initial joint density of (m, w) is thus $g_i(m|w)f_i(w)$ while $G_i(m|w)f_i(w)$ is the mass of workers of skill w with migration costs lower than m.

Following Mirrlees (1971), governments do not observe workers' types (w, m) and can only condition transfers on earnings y via an income tax function, $T_i(\cdot)$, for $i = A, B$. By assumption, taxes are levied according to the residence principle. In an open economy with international labor mobility, migration threat actually induces tax competition between these two governments, and we consider both Nash and Stackelberg competition (see Figure 1).

2.1 Individual Choices

Assume that all workers have the same additively separable utility function. So, for a worker of type (w, m) in country i:

$$u(c_i(w), l_i(w); \mu_i, \mu_{-i}, m) = v(c_i(w)) - h(l_i(w)) + \psi(\mu_i, \mu_{-i}) - \mathbb{I} \cdot m, \tag{1}$$

where c_i is consumption, l_i is labor (and $1 - l_i$ is leisure), $\mathbb{I}$ is equal to 1 if she decides to migrate and to 0 otherwise, μ_i is a domestic comparison consumption level, and μ_{-i} is a cross-country comparison consumption level, with $v' > 0 \geq v''$, $h' > 0$ and $h'' > 0$. Following common practice[2], comparison consumption levels are constructed as follows:

$$\mu_i = \int_{\underline{w}}^{\overline{w}} c_i(w) f_i(w) dw, \tag{2}$$

for $i \in \{A, B\}$. For later use, we give the following two assumptions.

Assumption 2.1 (Bounded Jealousy) $\max\{|\psi_i(\mu_i, \mu_{-i})|, |\psi_{-i}(\mu_i, \mu_{-i})|\} < v'(c_i(w))$ *for* $\psi_i(\mu_i, \mu_{-i}) \equiv \partial\psi/\partial\mu_i < 0$, $\psi_{-i}(\mu_i, \mu_{-i}) \equiv \partial\psi/\partial\mu_{-i} < 0$, $i \in \{A, B\}$ *and* $w \in [\underline{w}, \overline{w}]$.

Assumption 2.2 (Bounded Altruism) $\max\{\psi_i(\mu_i, \mu_{-i}), \psi_{-i}(\mu_i, \mu_{-i})\} < v'(c_i(w))$ *for* $\psi_i(\mu_i, \mu_{-i}) > 0$, $\psi_{-i}(\mu_i, \mu_{-i}) > 0$, $i \in \{A, B\}$ *and* $w \in [\underline{w}, \overline{w}]$.

Assumptions 2.1-2.2 state that the utility contribution of relative consumption is strictly smaller than that of absolute consumption, no matter the worker exhibits jealousy- or altruism-type relative consumption. These assumptions are consistent with general intuition as well as real data (see Clark et al., 2008).

The worker obtains her income from wages, with income denoted by $y_i \equiv w l_i(w) \geq 0$. Her budget constraint is thus:

$$c_i(w) = y_i(w) - T_i(y_i(w)). \tag{3}$$

Each worker is assumed to be small relative to the whole economy, and hence she takes μ_i and μ_{-i} as exogenously given. If she stays in country i, she maximizes (1) subject to $\mathbb{I} = 0$ and (3), yielding the first-order condition:

$$\frac{h'(l_i(w))}{w v'(c_i(w))} = 1 - T_i'(y_i(w)). \tag{4}$$

We denote by $U_i(w)$ her indirect utility. Using (4), the compensated and uncompensated elasticities of labor supply satisfy:

$$e^c(w_{n,i}) = \frac{h'(l_i)}{(h''(l_i) - w_{n,i}^2 v''(c_i)) l_i} > 0 \quad \text{and} \quad e^u(w_{n,i}) = \frac{h'(l_i) + w_{n,i}^2 v''(c_i) l_i}{(h''(l_i) - w_{n,i}^2 v''(c_i)) l_i} \tag{5}$$

[2]See, e.g., Oswald (1983), Kanbur and Tuomala (2013), and Aronsson and Johansson-Stenman (2015).

where $w_{n,i} \equiv w(1 - T_i'(y_i(w)))$ is the after-tax wage rate.

We now proceed to her migration decision. As is obvious, migration occurs if and only if $m < U_{-i}(w) - U_i(w)$. As in Lehmann et al. (2014), after combining the migration decisions made by workers born in both countries, the mass of residents of skill w in country i can be written as:

$$\phi_i(\Delta_i(w); w) \equiv \begin{cases} f_i(w) + G_{-i}(\Delta_i(w)|w) f_{-i}(w) n_{-i} & \text{for } \Delta_i(w) \geq 0, \\ (1 - G_i(-\Delta_i(w)|w)) f_i(w) & \text{for } \Delta_i(w) \leq 0. \end{cases} \tag{6}$$

with $\Delta_i(w) \equiv U_i(w) - U_{-i}(w)$. To ensure that $\phi_i(\cdot; w)$ is differentiable, we impose the technical restriction that $g_i(0|w)) f_i(w) = g_{-i}(0|w) f_{-i}(w) n_{-i}$, which is verified when the two countries are symmetric or when there is a fixed cost of migration, namely $g_i(0|w)) = g_{-i}(0|w) = 0$. We can then define the semi-elasticity of migration and the elasticity of migration, respectively, as:

$$\eta_i(\Delta_i(w); w) \equiv \frac{\partial \phi_i(\Delta_i(w); w)}{\partial \Delta_i} \frac{1}{\phi_i(\Delta_i(w); w)} \tag{7}$$

and

$$\theta_i(\Delta_i(w); w) \equiv c_i(w) \eta_i(\Delta_i(w); w). \tag{8}$$

For later use, and also to save on notations, we let $\tilde{f}_i(w) \equiv \phi_i(\Delta_i(w); w)$, $\tilde{\eta}_i(w) \equiv \eta_i(\Delta_i(w); w)$ and $\tilde{\theta}_i(w) \equiv \theta_i(\Delta_i(w); w)$.

2.2 Governments

In country $i \in \{A, B\}$, a benevolent government designs the tax system to maximize the welfare of the worst-off workers. By using (1) and (4), it is easy to show that $U_i(\underline{w}) = \min\{U_i(w) : w \in [\underline{w}, \overline{w}]\}$. That is, the worst-off are exactly those workers with wage rate $\underline{w}$ at the bottom of the skill distribution.

We choose maximin as the social objective due to the following considerations. First, many jobs of the workers of the lowest skills are at the bottom of global value chain and characterized as low-paid, insecure and dangerous (Gereffi and Luo, 2014). Second, they have the lowest migration (or foot-voting) ability, as migration rates increase in skill (Docquier and Marfouk, 2006). Third, especially for those in developed countries, the worst-off may be even worse in an open economy because they may lose jobs in the global competition with those workers of the lowest skills in developing countries. And fourth, as a normative criterion, maximin is a crucial principle in achieving the social justice suggested by Rawls (1971).

As is canonical, each government faces two sorts of constraints. The first is the fiscal budget constraint:

$$\int_{\underline{w}}^{\overline{w}} T_i(y_i(w))\phi_i(U_i(w) - U_{-i}(w); w)dw \geq R, \tag{9}$$

where $R \geq 0$ is an exogenous revenue requirement. As $v_c(\cdot) > 0$, (9) must be binding. In particular, here the participation constraint has been incorporated into the fiscal budget constraint (9) through the ex post skill density ϕ_i. The second is the set of incentive-compatibility constraints:

$$v(c_i(w)) - h(y_i(w)/w) \geq v(c_i(w')) - h(y_i(w')/w) \quad \forall w, w' \in [\underline{w}, \overline{w}]. \tag{10}$$

The necessary conditions for (10) to be satisfied are:

$$\dot{U}_i(w) = h'(l_i(w))\frac{l_i(w)}{w} \quad \forall w \in [\underline{w}, \overline{w}], \tag{11}$$

which gives the first-order incentive compatibility (FOIC) conditions. Sufficiency is guaranteed by the second-order incentive compatibility (SOIC) conditions, $\dot{y}_i(w) \geq 0$. If $\dot{y}_i(w) > 0$, then the first-order approach is appropriate.

As a result, optimal tax design is equivalent to solve the following maximization problem:

$$\max_{\{U_i(w), l_i(w), \mu_i\}} U_i(\underline{w})$$

subject to (2), (9), (11) and $\dot{y}_i(w) \geq 0$.

3 Nash Equilibrium

3.1 Optimal Tax Formula

We state the first major result in the following theorem.

Theorem 3.1 *In a Nash equilibrium with $\dot{y}_i(w) > 0$, the second-best marginal tax rates verify:*

$$\frac{T_i'(y_i(w))}{1 - T_i'(y_i(w))} = \overbrace{\frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}}^{\textit{Pigouvian-type tax}} + \overbrace{\mathcal{A}_i(w)\mathcal{B}_i(w)\mathcal{C}_i(w)}^{\textit{Mirrleesian-type tax}} \tag{12}$$

where: $\mathcal{A}_i(w) \equiv 1 + [l_i(w)h''(l_i)/h'(l_i)]$, $\mathcal{B}_i(w) \equiv \left[\tilde{F}_i(\overline{w}) - \tilde{F}_i(w)\right]/w\tilde{f}_i(w)$,

$$\mathcal{C}_i(w) \equiv \frac{v'(c_i(w))\int_w^{\overline{w}} \left\{\frac{1}{v'(c_i(t))}\left[1 + \frac{\gamma_i}{\lambda_i}\frac{f_i(t)}{\tilde{f}_i(t)}\right] - T_i(y_i(t))\tilde{\eta}_i(t)\right\}\tilde{f}_i(t)dt}{\tilde{F}_i(\overline{w}) - \tilde{F}_i(w)} \tag{13}$$

and

$$\frac{\gamma_i}{\lambda_i} = \frac{-\int_{\underline{w}}^{\overline{w}} \frac{\psi_i(\mu_i, \mu_{-i})}{v'(c_i(w))}\tilde{f}_i(w)dw}{1 + \int_{\underline{w}}^{\overline{w}} \frac{\psi_i(\mu_i, \mu_{-i})}{v'(c_i(w))}f_i(w)dw} \tag{14}$$

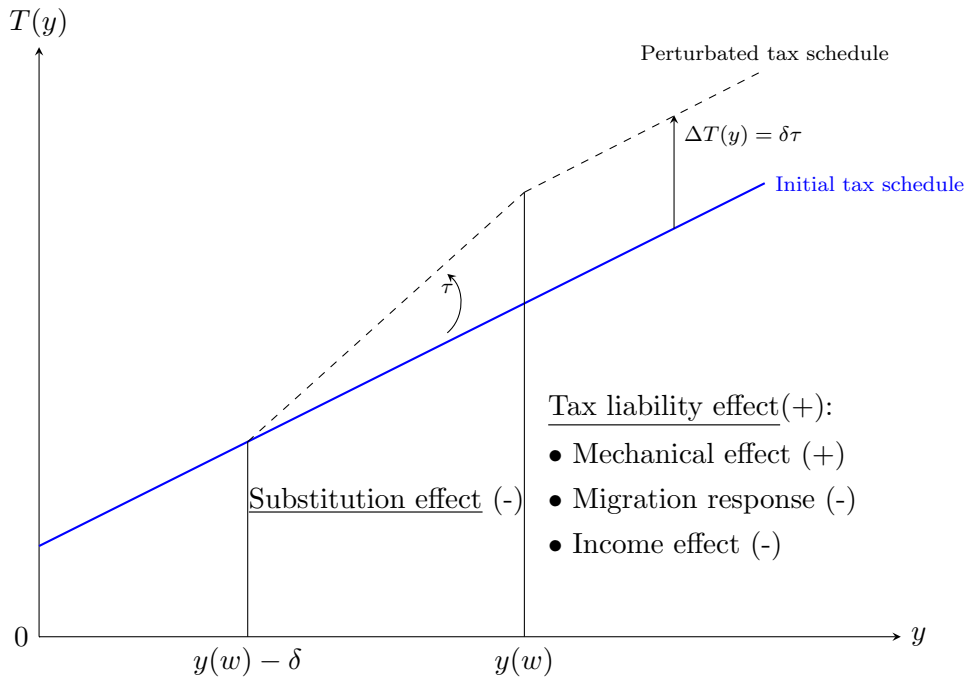

Figure 2: A Small Tax Reform Perturbation.

with $\tilde{F}_i(w) \equiv \int_{\underline{w}}^{w} \tilde{f}_i(t)dt$ *denoting the ex post skill distribution in country* $i \in \{A, B\}$*. Moreover, if* $T_i'(y_i(w))$ *is non-increasing in* w*, then the SOIC conditions are not binding, namely* $\dot{y}_i(w) > 0$ *holds.*

Proof. See Appendix. ■

Our optimal tax formula (12) differs from the classic one derived by Diamond (1998) and Saez (2001) in three ways: (i) the ex post mass $\tilde{f}_i(\cdot)$ of taxpayers replaces the ex ante density $f_i(\cdot)$, (ii) tax liability $T_i(y_i(\cdot))$ enters term $\mathcal{C}_i(w)$ of tax level effect, and (iii) there is a Pigouvian tax used to correct consumption externalities. Also, it differs from the one derived by K&T in two ways: (i) the ex post mass $\tilde{f}_i(\cdot)$ of taxpayers replaces the ex ante density $f_i(\cdot)$, and (ii) tax liability $T_i(y_i(\cdot))$ enters term $\mathcal{C}_i(w)$ of tax level effect. As is clear soon, these differences indeed lead to much more challenging qualitative and quantitative analyses than what we have seen in existing literature.

To intuitively interpret the optimal tax formula (12), we investigate the effects of a small tax reform in a unilaterally deviating country i: the second-best marginal tax rates $T_i'(y_i(w))$ are uniformly increased by a small amount $\tau > 0$ on a small income interval $[y_i(w) - \delta, y_i(w)]$ as shown in Figure 2 for some small constant $\delta > 0$. As a consequence, tax liabilities above $y_i(w)$ are uniformly increased by $\delta\tau$. This gives rise to the following effects.

First, a worker with income in $[y_i(w)-\delta, y_i(w)]$ responds to the rise in the marginal tax rate by a substitution effect between leisure and labor, which hence reduces the taxes she pay. Second, each worker with skills above w faces a lump-sum increase $\delta\tau$ in her tax liability, which is called mechanical effect in the literature (e.g., Saez, 2001). Since the unilateral rise in tax liability reduces her indirect utility in the deviating country, compared to its competitor, the number of labor outflow increases and hence the number of taxpayers with skills above w decreases. Following Lehmann et al. (2014), we define the tax liability effect as the sum of the mechanical and migration effects for all skill levels above w. And third, the increase in tax liability tightens the consumption budget, and hence it follows from (13) that income effect will in turn reduce the positive mechanical effect. Since the optimal tax formula (12) is derived based on the Nash equilibrium, hence any unilateral deviation we consider cannot induce any first-order effect on the tax revenues of the deviating country i. This implies that the tax liability effect must be positive so that the substitution effect is offset by the tax liability effect.

To see how relativity changes the average tax rate (ATR)[3] and marginal tax rate (MTR), we also numerically solve the optimal tax formula (12) under the following assumptions (see Figure 3). First, the two countries are assumed to symmetric. Second, following Jacquet et al. (2013), we put the mode $w_m = \$19,800$ and the highest skill level $\overline{w} = \$40,748$ with workers within this income interval having a Pareto income distribution, with density function $f(w) = aw_m^a/w^{a+1}$ for $w_m \leq w \leq \overline{w}$.[4] Third, we use quasi-linear in consumption preferences with a constant elasticity of labor supply, namely $u_i = c_i - l_i^{1+\frac{1}{\varepsilon}}/\left(1+\frac{1}{\varepsilon}\right) + \sigma_D\mu_i + \sigma_F\mu_{-i}$ with $\sigma_D, \sigma_F \in (-1,0)$. And fourth, also similar to the distribution assumption used by Jacquet et al. (2013), we let the conditional distribution of migration costs be logistic:

$$G(0|w) = \frac{\exp(-\chi w)}{1+\exp(-\chi w)} \quad \text{for} \quad \chi \in (0,1).$$

Parameter values for simulation are given by $\varepsilon = 0.25$, $a = 2$, $\chi = 0.5$, $l = 0.33$ and $\sigma_D \in (-1,0)$. It follows from Figure 3 that both ATR and MTR increase as the degree of relative consumption concern $|\sigma_D|$ increases, for any $w \in [w_m, \overline{w}]$.

[3]Since it is impossible to solve for a formula of ATR in the current context, we rely on numerical simulation to see the shape of ATR and how it changes with respect to the change of the degree of consumption relativity.

[4]For the U.S. between 1913-1998, Piketty and Saez (2003) estimate the Pareto index to be $a = 1.5$ for the upper part of income distribution. However, recently Diamond and Saez (2011) estimate the Pareto index to be 2 for the U.S. economy. In this simulation, we choose $a = 2$.

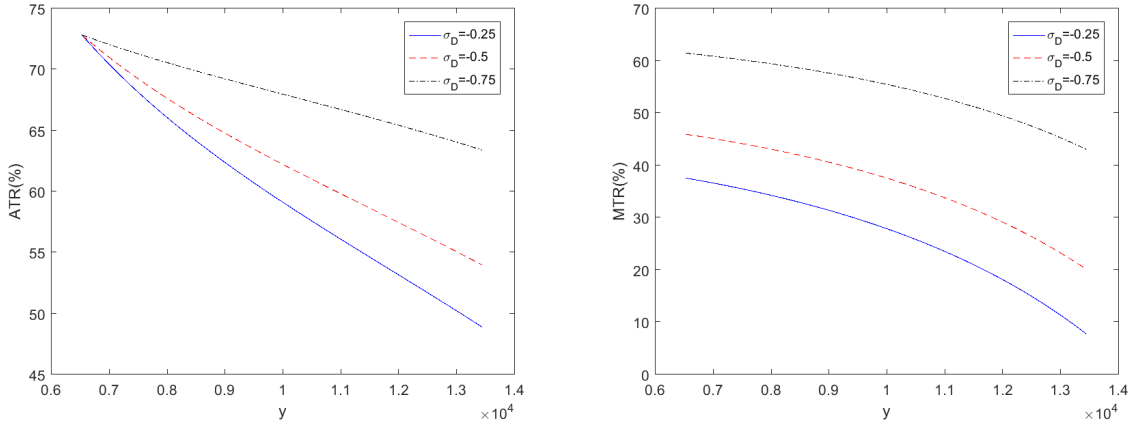

Figure 3: ATR and MTR in a Symmetric Nash equilibrium.

3.2 Qualitative Properties

To derive the qualitative properties of the optimal tax formula established in Theorem 3.1, we follow the approach developed by Jacquet et al. (2013) and start by considering the same problem as in the second best, except that skills w are common knowledge, so migration costs m remain private information. Using the same terminology as Lehmann et al. (2014), we call this benchmark the Tiebout best.

Lemma 3.1 *In a Nash equilibrium, we have the following predictions:*

(i) The Tiebout-best tax liabilities are given by

$$T_i^*(y_i(w)) = \frac{1}{v'(c_i(w))\tilde{\eta}_i(w)}\left[1 + \frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}\right]$$

for $\forall i \in \{A, B\}$, $\forall w \in (\underline{w}, \overline{w}]$, with an upward jump discontinuity at $\underline{w}$.

(ii) The Tiebout-best marginal tax rates verify:

$$\frac{T_i^{*\prime}(y_i(w))}{1 - T_i^{*\prime}(y_i(w))} = \frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)} \quad \forall w \in [\underline{w}, \overline{w}]$$

with γ_i/λ_i given in Theorem 3.1.

Proof. See Appendix. ■

Under jealousy-type consumption comparison, it follows from (14) that $\gamma_i/\lambda_i > 0$. So for all skills but the bottom skill, the Tiebout-best tax liabilities under jealousy are strictly decreasing in the elasticity of migration, as shown in part (i). In addition, if the revenue requirement R is sufficiently small, then it follows from the fiscal-budget constraint (9) that the worst-off workers receive net transfers in the Tiebout-best economy. However, we have $\gamma_i/\lambda_i < 0$

under altruism-type consumption comparison, so the Tiebout-best tax liabilities are strictly decreasing in the elasticity of migration and the worst-off workers receive net transfers only when $\tilde{f}_i(w)/f_i(w) > -\gamma_i/\lambda_i$, namely either the amount of labor flow is bounded below or the degree of consumption comparison is bounded above. As shown in part (ii), the Tiebout-best marginal tax rates are used for correcting consumption externality as well as attracting labor inflow. In particular, tax rates are strictly positive under jealousy while strictly negative under altruism. Also, the ex ante to ex post density ratio $f_i(w)/\tilde{f}_i(w)$ and jealousy comparison impose a complementary effect while this ratio and altruism comparison impose a substitutive effect on the Tiebout-best tax liabilities and tax rates.

The following proposition gives a complete characterization of the second-best tax schedules under jealousy-type consumption comparison.

Proposition 3.1 *Suppose Assumption 2.1 holds, then the optimal tax structure in the Nash equilibrium has the following characteristics:*

(i) If $T_i(y_i(w)) \leq T_i^*(y_i(w))$ *for* $\forall w \in (\underline{w}, \overline{w})$*, then* $T_i'(y_i(w)) > 0$ *for* $\forall w \in (\underline{w}, \overline{w})$*.*

(ii) $T_i'(y_i(\underline{w})) > T_i^{*'}(y_i(\underline{w})) > 0$ *and* $T_i'(y_i(\overline{w})) = T_i^{*'}(y_i(\overline{w})) > 0$ *for* $\overline{w} < \infty$*.*

(iii) If $h(\cdot)$ *is isoelastic,* $f_i(w)/\tilde{f}_i(w)$ *is decreasing in* w*, and* $T_i(y_i(w)) \leq T_i^*(y_i(w))$ *for* $\forall w \in [\underline{w}, \overline{w}]$*, then we have:*

(a) $T_i'(y_i(w))$ *is decreasing for* $w \leq w_m$*; and*

(b) $T_i'(y_i(w))$ *is decreasing for* $w > w_m$ *when* $wf_i(w)$ *is non-decreasing in* w*.*

(iv) If $f_i(w)/\tilde{f}_i(w)$ *is non-increasing in* w*,* $-\frac{v''(c_i(w))\dot{c}_i(w)}{v'(c_i(w))} \leq \frac{\dot{\tilde{\eta}}_i(w)}{\tilde{\eta}_i(w)}$*, and there exists a* $\tilde{w} \in (\underline{w}, \overline{w})$ *such that* $T_i'(y_i(\tilde{w})) \geq 0$*, then*

$$T_i(y_i(w)) < \begin{cases} T_i^*(y_i(w)) & \text{for } \underline{w} < w \leq \tilde{w}, \\ \frac{1}{v'(c_i(w))\tilde{\eta}_i(w)}\left[1 + \frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}\right] & \text{for } w = \underline{w}. \end{cases}$$

Proof. See Appendix. ■

Parts (i)-(ii) identify (sufficient) conditions such that the second-best marginal tax rates are strictly positive over the entire income distribution. In particular, if tax liabilities are bounded above by the Tiebout-best tax liabilities, then the second-best marginal tax rates are strictly positive for almost all skills. Part (iii) identifies conditions such that the SOIC conditions are not binding, namely the first-order approach is reliable. Part (iv) identifies a sufficient condition such that the second-best tax liabilities are bounded above by the Tiebout-best tax liabilities.

The following proposition gives a complete characterization of the second-best tax schedules under altruism-type consumption comparison.

Proposition 3.2 *Suppose Assumption 2.2 holds, then the optimal tax structure in the Nash equilibrium has the following characteristics:*

(i) $T_i'(y_i(\underline{w})) > T_i^{*'}(y_i(\underline{w}))$ *and* $T_i'(y_i(\overline{w})) = T_i^{*'}(y_i(\overline{w})) < 0$ *for* $\overline{w} < \infty$.

(ii) Let $\Gamma_i(w) \equiv 1 + \frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}$. *If* $\Gamma_i(w) \geq 0$ *and* $\dot{\Gamma}_i(w) \geq \Gamma_i(w)\frac{\dot{\tilde{\eta}}_i(w)}{\tilde{\eta}_i(w)}$, *then we have*

(a) $\mathcal{A}_i(w)\mathcal{B}_i(w)\mathcal{C}_i(w) > 0$ *for* $\forall w \in (\underline{w}, \overline{w})$;

(b) $T_i(y_i(\overline{w})) < T_i^*(y_i(\overline{w}))$;

(c) $T_i(y_i(\underline{w})) > \Gamma_i(\underline{w})/[v'(c_i(\underline{w}))\tilde{\eta}_i(\underline{w})]$.

Proof. See Appendix. ■

For the worst-off workers, the second-best marginal tax rates under altruism comparison are strictly higher than the corresponding Tiebout-best marginal tax rates. For the top-income workers with bounded skill distribution, the second-best marginal tax rates under altruism comparison are equal to the corresponding Tiebout-best marginal tax rates and are strictly negative. Part (ii) identifies mild conditions such that the following conclusions hold: (a) the Mirrleesian-type tax in the optimal tax formula (12) is strictly positive for all skills but the two end points; (b) for top-income workers, the second-best tax liabilities are strictly smaller than the Tiebout-best tax liabilities; and (c) there is a lower bound of the second-best tax liabilities for the worst-off workers.

The following proposition establishes a closed-form formula of the optimal asymptotic tax rates (or the tax rates placed on top-income workers) under certain conditions.

Proposition 3.3 *Suppose economic environments satisfy the following conditions:*

(a) $v'(\cdot) = 1$, *namely quasi-linear in consumption preferences;*

(b) $h(\cdot)$ *is isoelastic with elastic coefficient* $\varepsilon > 0$;

(c) $F_i(w)$ *is a Pareto distribution with* $\overline{w} = \infty$ *and Pareto index* $a_i > 1$.

Then, the optimal asymptotic marginal tax rate (AMTR) in a Nash equilibrium is:

$$T_i'(y_i(\infty)) = \frac{\frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + \left[1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty)\right](1+\varepsilon)(1/a_i)}{1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + \left[1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + \tilde{\theta}_i(\infty)\right](1+\varepsilon)(1/a_i)},$$

with $\tilde{\theta}_i(\infty) \equiv \lim_{w\uparrow\infty}\tilde{\theta}_i(w) \geq 0$ *and* $\alpha_i(\infty) \equiv \lim_{w\uparrow\infty}\frac{f_i(w)}{\tilde{f}_i(w)} \geq 0$.

Proof. See Appendix. ■

Given that the optimal tax formula (12) is quite complicated, restrictions (a)-(c) must be tolerated for explicitly solving for the optimal AMTR. In fact, conditions (a)-(b) are widely used in the literature of optimal taxation, and Pareto distribution is an empirically relevant assumption for high-income workers. In the current context, the optimal AMTR is a continuously differentiable function of five important variables: the degree of consumption comparison γ_i/λ_i, the measure of labor flow $\alpha_i(\infty)$, the elasticity of labor supply ε, the degree of income inequality $1/a_i$, and the elasticity of migration $\tilde{\theta}_i(\infty)$. In particular, it is straightforward that AMTR is strictly decreasing in the elasticity of migration.

The following two propositions characterize the composition effect of consumption relativity and income inequality on the optimal AMTR. The composition effect can be completely different under alternative circumstances.

Proposition 3.4 *Suppose* $\psi(\mu_i, \mu_{-i}) = \sigma_D \mu_i + \tilde{\psi}(\mu_{-i})$ *for a constant* $\sigma_D \in (-1, 0)$, $F_i(w) = F_{-i}(w)$ *and* $\partial \tilde{F}_i(\infty)/\partial a_i = 0$, *then we have the following predictions.*

(i) If $\tilde{\theta}_i(\infty) \leq 1$, *then* $\frac{\partial^2 T_i'(y_i(\infty))}{\partial(-\sigma_D)\partial(1/a_i)} < 0$.

(ii) If $\tilde{\theta}_i(\infty) > 1$, *then*

$$\frac{\partial^2 T_i'(y_i(\infty))}{\partial\left(-\sigma_D\right)\partial(1/a_i)} \begin{cases} < 0 & \text{for } \alpha_i(\infty) < \left(\frac{\lambda_i}{\gamma_i}\right) \frac{1+\tilde{\theta}_i(\infty)+\frac{1+\varepsilon}{a_i}\left[1+\tilde{\theta}_i(\infty)^2\right]}{\left(1+\frac{1+\varepsilon}{a_i}\right)\left[\tilde{\theta}_i(\infty)-1\right]}; \\ > 0 & \text{for } \alpha_i(\infty) > \left(\frac{\lambda_i}{\gamma_i}\right) \frac{1+\tilde{\theta}_i(\infty)+\frac{1+\varepsilon}{a_i}\left[1+\tilde{\theta}_i(\infty)^2\right]}{\left(1+\frac{1+\varepsilon}{a_i}\right)\left[\tilde{\theta}_i(\infty)-1\right]}. \end{cases}$$

Proof. See Appendix. ■

Proposition 3.4 analyzes the case with jealousy-type consumption comparison. If the elasticity of migration is not greater than one, then relativity and inequality play substitutive roles in shaping AMTR. Precisely, the higher is inequality, the lower is the effect of relativity in raising AMTR; similarly, the higher is relativity, the lower is the effect of inequality in raising AMTR. However, if the elasticity of migration is greater than one, then relativity and inequality play substitutive roles only when the ex post mass of top-income workers is greater than a threshold, otherwise relativity and inequality play complementary roles in shaping the optimal AMTR.

Under similar assumptions, K&T show that relativity and inequality always play substitutive roles in a closed economy. Here we show that such a conclusion depends on the elasticity of migration and the level of migration in an open economy. Therefore, Proposition 3.4 generalizes the prediction of K&T as a special case with $\tilde{\theta}_i(\infty) = 0$.

Proposition 3.5 *Suppose* $\psi(\mu_i, \mu_{-i}) = \sigma_D \mu_i + \tilde{\psi}(\mu_{-i})$ *for a constant* $\sigma_D \in (0,1)$, $F_i(w) = F_{-i}(w)$ *and* $\partial \tilde{F}_i(\infty)/\partial a_i = 0$, *then we have the following predictions.*

(i) If $\tilde{\theta}_i(\infty) < 1$, *then*

$$\frac{\partial^2 T_i'(y_i(\infty))}{\partial \sigma_D \partial(1/a_i)} \begin{cases} < 0 & \text{for } \left(-\frac{\lambda_i}{\gamma_i}\right) \frac{1+\frac{1+\varepsilon}{a_i}\left[1+\tilde{\theta}_i(\infty)\right]}{1+\frac{1+\varepsilon}{a_i}} < \alpha_i(\infty) < \left(-\frac{\lambda_i}{\gamma_i}\right) \frac{1+\tilde{\theta}_i(\infty)+\frac{1+\varepsilon}{a_i}\left[1+\tilde{\theta}_i(\infty)^2\right]}{\left(1+\frac{1+\varepsilon}{a_i}\right)\left[1-\tilde{\theta}_i(\infty)\right]}; \\ > 0 & \text{otherwise.} \end{cases}$$

(ii) If $\tilde{\theta}_i(\infty) = 1$, *then*

$$\frac{\partial^2 T_i'(y_i(\infty))}{\partial \sigma_D \partial(1/a_i)} \begin{cases} < 0 & \text{for } \alpha_i(\infty) > \left(-\frac{\lambda_i}{\gamma_i}\right) \frac{1+2\left(\frac{1+\varepsilon}{a_i}\right)}{1+\frac{1+\varepsilon}{a_i}}; \\ > 0 & \text{for } \alpha_i(\infty) < \left(-\frac{\lambda_i}{\gamma_i}\right) \frac{1+2\left(\frac{1+\varepsilon}{a_i}\right)}{1+\frac{1+\varepsilon}{a_i}}. \end{cases}$$

(iii) If $\tilde{\theta}_i(\infty) > 1$, *then*

$$\frac{\partial^2 T_i'(y_i(\infty))}{\partial \sigma_D \partial(1/a_i)} \begin{cases} < 0 & \text{for } \alpha_i(\infty) > \left(-\frac{\lambda_i}{\gamma_i}\right) \frac{1+\frac{1+\varepsilon}{a_i}\left[1+\tilde{\theta}_i(\infty)\right]}{1+\frac{1+\varepsilon}{a_i}}; \\ > 0 & \text{for } \alpha_i(\infty) < \left(-\frac{\lambda_i}{\gamma_i}\right) \frac{1+\frac{1+\varepsilon}{a_i}\left[1+\tilde{\theta}_i(\infty)\right]}{1+\frac{1+\varepsilon}{a_i}}. \end{cases}$$

Proof. See Appendix. ■

Proposition 3.5 analyzes the composition effect of relativity and inequality on the optimal AMTR under altruism-type consumption comparison. Compared to the case with jealousy-type consumption comparison, the current predictions seem to be more subtle.

If the elasticity of migration is smaller than one, then relativity and inequality impose substitutive effect on AMTR only when the ex post mass of top-income workers is bounded both below and above. Otherwise, they impose complementary effect, namely the higher is inequality, the higher is the effect of relativity in raising AMTR; or the higher is relativity, the higher is the effect of inequality in raising AMTR. If the elasticity of migration is greater than or equal to one, then relativity and inequality impose substitutive effect on AMTR only when the ex post mass of top-income workers is smaller than some threshold, and this threshold is different for different values of migration elasticity.

4 Stackelberg Equilibrium

4.1 Optimal Tax Formula

Without any loss of generality, we denote by i the leader country and $-i$ the follower country in the current Stackelberg game. We thus state the second major result in the following theorem.

Theorem 4.1 *In a Stackelberg equilibrium, the optimal tax formula is the same as that in the Nash equilibrium, except that:*

$$\frac{\gamma_i}{\lambda_i} = \frac{\int_{\underline{w}}^{\overline{w}} \left(\frac{\partial c_i(w)}{\partial \mu_i} + \frac{\partial c_i(w)}{\partial \mu_{-i}} \frac{\partial \mu_{-i}}{\partial \mu_i} \right) \tilde{f}_i(w) dw}{1 - \int_{\underline{w}}^{\overline{w}} \left(\frac{\partial c_i(w)}{\partial \mu_i} + \frac{\partial c_i(w)}{\partial \mu_{-i}} \frac{\partial \mu_{-i}}{\partial \mu_i} \right) f_i(w) dw}$$

with

$$\frac{\partial \mu_{-i}}{\partial \mu_i} = \frac{\int_{\underline{w}}^{\overline{w}} \frac{\partial c_{-i}(w)}{\partial \mu_i} f_{-i}(w) dw}{1 - \int_{\underline{w}}^{\overline{w}} \frac{\partial c_{-i}(w)}{\partial \mu_{-i}} f_{-i}(w) dw}$$

for the leader country i.

Proof. See Appendix. ■

Theorems 4.1 and 3.1 together demonstrate how the form of tax competition might affect the optimal tax rates. Intuitively, since the leader country takes into account the behavioral response of the follower country in the dynamic Stackelberg game, it partially internalizes cross-country consumption externalities, namely the additional term $\partial \mu_{-i} / \partial \mu_i$ is in general different from zero.

4.2 Qualitative Properties

Using Theorem 4.1, the following corollary is immediate.

Corollary 4.1 *If $|\psi_i(\mu_i, \mu_{-i})| > |\psi_{-i}(\mu_i, \mu_{-i})|$ for country i, then the qualitative properties (of Nash equilibrium) established in Propositions 3.1-3.3 carry over to the current Stackelberg equilibrium.*

For additively separable functional forms of $\psi(\mu_i, \mu_{-i})$, condition $|\psi_i(\mu_i, \mu_{-i})| > |\psi_{-i}(\mu_i, \mu_{-i})|$ means that the degree of domestic consumption comparison is greater than that of cross-country consumption comparison for workers in country i. Given the real-life observation that people are more often to make status comparison with people who live in their social networks, this restriction can be regarded as reasonable.

Proposition 4.1 *If economic environments satisfy the following conditions:*

(a) The utility function of relative consumptions has the form:

$$\psi(\mu_i, \mu_{-i}) = \begin{cases} \sigma_D \mu_i + \sigma_F \mu_{-i} & \text{for country } i, \\ \sigma_D \mu_{-i} + \sigma_F \mu_i & \text{for country } -i \end{cases}$$

with coefficients $\sigma_D, \sigma_F \in (-1, 0) \cup (0, 1)$ and $|\sigma_F| + |\sigma_D| < 1$;

(b) $F_i(w) = F_{-i}(w)$*;*

(c) $\partial \tilde{F}_i(\infty)/\partial a_i = 0$.

Then, for the optimal AMTR of leader country i *in a Stackelberg equilibrium, the predictions established in Propositions 3.4-3.5 carry over to the current equilibrium.*

Proof. See Appendix. ■

Provided that we have assumed quasi-linear in consumption preferences in solving for the optimal AMTR, condition (a) is thus a natural restriction. Condition (b) simplifies our analysis by eliminating asymmetry between these two countries, which however is not an essential requirement for establishing the current prediction. Condition (c) is a technical assumption mainly for the purpose of simplicity. The main message implied by Proposition 4.1 is that the composition effect of relativity and inequality imposed on the optimal AMTR is in general the same under both forms of tax competition, even though the corresponding AMTRs are in general different.

Proposition 4.2 *If Assumption 2.1 holds, then the government of the leader country imposes a higher marginal tax rate in the Stackelberg equilibrium than that in the Nash equilibrium.*

Proof. See Appendix. ■

Intuitively, since jealousy implies negative consumption externality and marginal tax rates strictly increase as externality increases, the leader country who (partially) internalizes cross-country consumption externalities imposes a higher tax rate than that it may impose in a simultaneous-move static game where no one internalizes cross-country consumption externalities.

5 Numerical Illustration

In this section we provide some numerical examples on the optimal AMTR established in Proposition 3.3. These numerical experiments enable us to see quantitatively how large the difference on the optimal AMTR can be made by the effects of strategic tax competition and cross-country consumption comparisons, when compared to what K&T have obtained in a closed economy through completely ignoring these effects.

For simplicity, we use the linear utility function of relative consumptions shown in condition (a) of Proposition 4.1. The following tables present AMTRs for different parameter values, when the Pareto index $a_i = 2$ and 3, the coefficient of domestic relative consumption $\sigma_D = -0.5$, 0 and 0.5, and the elasticity of labor supply $\varepsilon = 0.25$, 0.33, and 0.5. We consider three elasticity

Table 1: Parameter Values

	Value	Description	Source/Target
$\tilde{\theta}_i(\infty)$	0.25	Migration elasticity	Piketty & Saez (2013)
ε	(0.1,0.4)	Labor-supply elasticity	Saez et al. (2012)
a_i	2	Pareto index	Diamond & Saez (2011)
σ_D	-0.5	Domestic relativity	Clark et al. (2008)
σ_F	-0.2	Cross-country relativity	$\lvert\sigma_F\rvert < \lvert\sigma_D\rvert$

scenarios. The first two with $\varepsilon = 0.25$ and 0.33 are realistic midrange estimates (see Saez et al., 2012), while $\varepsilon = 0.5$ is a little bit larger than the current average empirical estimates. We consider two inequality scenarios. The first one with $a_i = 2$ is based on the 2005 U.S. empirical income distribution (see Diamond and Saez, 2011), while $a_i = 3$ is chosen to be larger than this realistic number to represent an experimental scenario with more equal income distribution.

Moreover, we consider three relativity scenarios. $\sigma_D = 0$ denotes the benchmark case without relative consumption, whereas $\sigma_D = -0.5$ measures the degree of jealousy and $\sigma_D = 0.5$ measures the degree of altruism. One of the key findings of the empirical research on relativity is that the estimated coefficient on income (consumption) and income comparison is statistically almost equal and opposite (see, e.g., Luttmer, 2005). Given the assumption of quasi-linear preferences, $\sigma_D = -0.5$ seems to be reasonable. In fact, it is consistent with the finding of Alpizar et al. (2005) who use survey-experimental methods to see how much we care about absolute versus relative income and consumption. We chose $\sigma_D = 0.5$ simply for the sake of comparison with the case with $\sigma_D = -0.5$. By assumption, the degree of cross-country social comparisons is smaller than that of domestic social comparisons, so we let $\sigma_F^2 = 0.04$ in what follows. Following Piketty and Saez (2013), we let the value of the elasticity of migration to be 0.25, i.e., $\tilde{\theta}_i(\infty) = 0.25$. We summarize all realistic parameter values in Table 1.

If $\Delta_i(\infty) \geq 0$, namely top-income workers get an indirect utility in country i which is not less than that they can get in the other country $-i$, then the density ratio $\alpha_i(\infty)$ must not be greater than 1. That is, country i has the potential to attract more high-skill workers from the opponent country $-i$. Similarly, if $\Delta_i(\infty) \leq 0$, then the density ratio $\alpha_i(\infty)$ must not be smaller than 1. The following tables consider both cases.

Table 2: A Summary of Quantitative Findings

		Nash	Nash	Stack	Stack	
	Relativity effect	=	=	>	>	
	Migration effect	Small	Large	Small	Large	
MTR^O	Labor inflow	<	<<	>	<<	MTR^C
MTR^O	Labor outflow	<	>>	>	>>	MTR^C

5.1 A Comparison with K&T

In Tables 3-10, we use red numbers to denote the optimal AMTRs calculated using the formula of K&T. In both types of equilibrium, we obtain the following findings under different values of $\alpha_i(\infty)$. In particular, negative numbers imply that workers receive transfers, which occurs only when workers exhibit altruism-type relative consumption preferences.

First, for each given labor-supply elasticity and given degree of relative consumption, AMTR increases as inequality increases. Second, for each given degree of inequality and given degree of relative consumption, AMTR increases as elasticity increases. And third, for each given elasticity and given degree of inequality, AMTR significantly increases under jealousy and significantly declines under altruism, compared to the benchmark case without relative consumption concerns.

We summarize major quantitative findings in Table 2, in which the superscripts of MTR^O and MTR^C denote open-economy and closed-economy, respectively. In particular, we just consider the MTR of the leader country under Stackelberg tax competition. Essentially, as already shown in Table 2, relativity and migration are determinant factors in such a comparison.

Since no one internalizes the cross-country consumption externality under Nash competition, the relativity effect on MTR is the same between open economy and closed economy. In contrast, as the leader country internalizes cross-country consumption externality under Stackelberg competition, the relativity effect on MTR implemented by the leader country in an open economy should be greater than that in a closed economy. Therefore, if there is no migration between countries, only the MTR implemented under Stackelberg competition should be higher than that implemented in a closed economy.

For comparing the Nash MTR^O and MTR^C, migration effect definitely dominates relativity effect. If labor flow is small, no matter it is inflow or outflow, Nash competition implies a smaller MTR than that in a closed economy without any migration threat imposed on the government. Nevertheless, if labor flow is large, then migration effect is heterogenous between the case with labor inflow and the case with labor outflow. Precisely, large labor inflow must be induced by

a much lower MTR compared to MTR^C, while large labor outflow must be induced by a much higher MTR compared to MTR^C.

For comparing the Stackelberg MTR^O and MTR^C, both relativity effect and migration effect matter. If labor flow is small, then relativity effect strictly dominates migration effect for both the case with labor inflow and the case with labor outflow, implying that MTR^O under Stackelberg competition should be higher than MTR^C. However, if labor flow is large, then migration effect strictly dominates relativity effect and it is heterogenous between the case with labor inflow and the case with labor outflow. Precisely, large labor inflow must be induced by a much lower MTR compared to MTR^C, while large labor outflow must be induced by a much higher MTR compared to MTR^C. As a result, under large labor flow, the prediction is the same between Nash and Stackelberg tax competition.

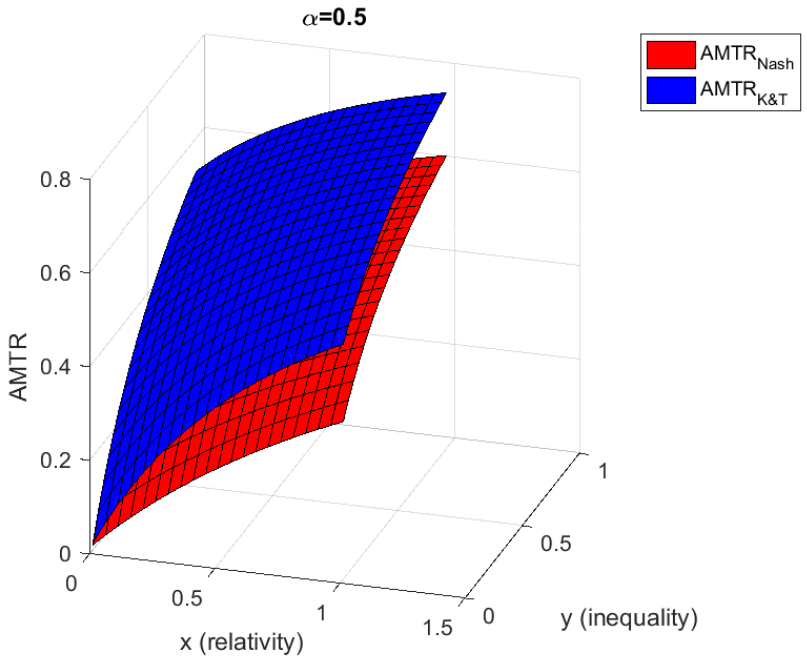

Figure 4: Parameters: $\alpha_i(\infty) = 0.5$, $\varepsilon = \tilde{\theta}_i(\infty) = 0.25$, $a_i > 1$, and $\sigma_D \in (-1, 0)$.

5.1.1 Nash vs. K&T

Tables 3-6 compare optimal AMTRs in Nash equilibrium with those in K&T. They show that the difference on AMTRs increases as the net level of migration increases, precisely as $\alpha_i(\infty)$ declines under $\Delta_i(\infty) \geq 0$ and as $\alpha_i(\infty)$ increases under $\Delta_i(\infty) \leq 0$. Under jealousy-type relativity with $\Delta_i(\infty) \geq 0$, the AMTRs under Nash equilibrium are always smaller than those in K&T (see Figure 4). However, if $\Delta_i(\infty) \leq 0$, the AMTRs under Nash equilibrium are

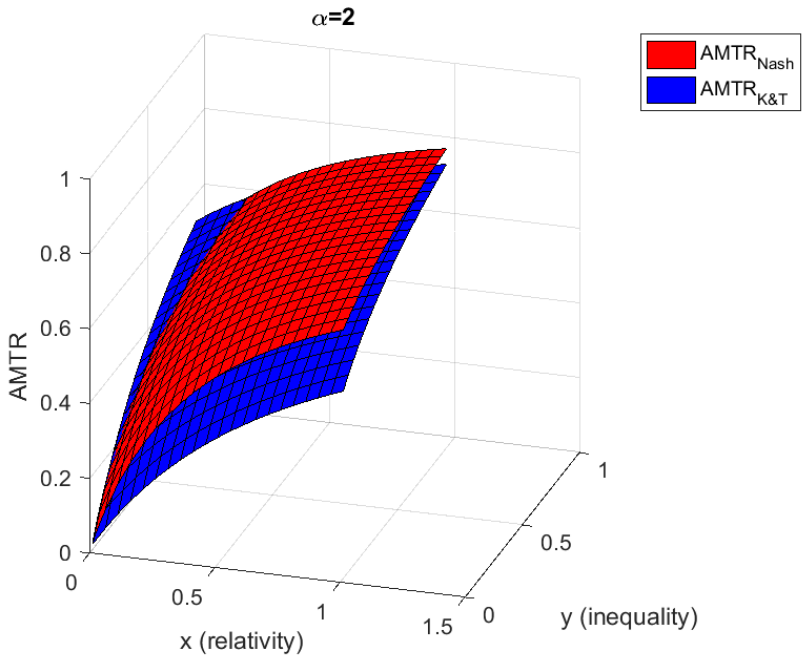

Figure 5: Parameters: $\alpha_i(\infty) = 2$, $\varepsilon = \tilde{\theta}_i(\infty) = 0.25$, $a_i > 1$, and $\sigma_D \in (-1, 0)$.

greater than those in K&T when $\alpha_i(\infty)$ is larger than some critical value (see Figure 5). Under altruism-type relativity with $\Delta_i(\infty) \geq 0$, the AMTRs under Nash equilibrium are always greater than those in K&T. However, if $\Delta_i(\infty) \leq 0$, the AMTRs under Nash equilibrium are always smaller than those in K&T. Also, if there is no relative consumption concern, then the AMTRs under Nash equilibrium are always smaller than those in K&T. Moreover, the migration effect is magnified by consumption comparison.

Table 3: AMTR (%) under $\Delta_i(\infty) \geq 0$ with $\alpha_i(\infty) = 0.95$

	$\varepsilon = 0.25$	$\varepsilon = 0.25$	$\varepsilon = 0.33$	$\varepsilon = 0.33$	$\varepsilon = 0.5$	$\varepsilon = 0.5$
	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$
$\sigma_D = -0.5$	65.2,69.2	61.5,64.7	65.8,70.0	62.0,65.4	67.0,71.4	63.1,66.7
$\sigma_D = 0$	35.1,38.5	27.4,29.4	36.3,39.9	28.5,30.7	38.7,42.9	30.8,33.3
$\sigma_D = 0.5$	8.7,7.7	-3.0,-5.9	10.6,9.9	-1.3,-3.9	14.2,14.3	2.2,0.0

5.1.2 Stackelberg vs. K&T

Tables 7-10 compare optimal AMTRs in Stackelberg equilibrium, denoted by blue numbers, with those in K&T. The difference on AMTRs increases as the net level of migration increases, precisely as $\alpha_i(\infty)$ declines under $\Delta_i(\infty) \geq 0$ and as $\alpha_i(\infty)$ increases under $\Delta_i(\infty) \leq 0$. Under

Table 4: AMTR (%) under $\Delta_i(\infty) \geq 0$ with $\alpha_i(\infty) = 0.67$

	$\varepsilon = 0.25$	$\varepsilon = 0.25$	$\varepsilon = 0.33$	$\varepsilon = 0.33$	$\varepsilon = 0.5$	$\varepsilon = 0.5$
	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$
$\sigma_D = -0.5$	59.7,69.2	55.3,64.7	60.4,70.0	55.9,65.4	61.8,71.4	57.2,66.7
$\sigma_D = 0$	35.1,38.5	27.4,29.4	36.3,39.9	28.5,30.7	38.7,42.9	30.8,33.3
$\sigma_D = 0.5$	18.5,7.7	8.3,-5.9	20.1,9.9	9.8,-3.9	23.2,14.3	12.8,0.0

Table 5: AMTR (%) under $\Delta_i(\infty) \leq 0$ with $\alpha_i(\infty) = 1.05$

	$\varepsilon = 0.25$	$\varepsilon = 0.25$	$\varepsilon = 0.33$	$\varepsilon = 0.33$	$\varepsilon = 0.5$	$\varepsilon = 0.5$
	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$
$\sigma_D = -0.5$	66.8,69.2	63.3,64.7	67.4,70.0	63.8,65.4	68.5,71.4	64.8,66.7
$\sigma_D = 0$	35.1,38.5	27.4,29.4	36.3,39.9	28.5,30.7	38.7,42.9	30.8,33.3
$\sigma_D = 0.5$	4.6,7.7	-7.7,-5.9	6.6,9.9	-5.9,-3.9	10.4,14.3	-2.2,0.0

Table 6: AMTR (%) under $\Delta_i(\infty) \leq 0$ with $\alpha_i(\infty) = 1.55$

	$\varepsilon = 0.25$	$\varepsilon = 0.25$	$\varepsilon = 0.33$	$\varepsilon = 0.33$	$\varepsilon = 0.5$	$\varepsilon = 0.5$
	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$
$\sigma_D = -0.5$	73.1,69.2	70.3,64.7	73.6,70.0	70.7,65.4	74.5,71.4	71.5,66.7
$\sigma_D = 0$	35.1,38.5	27.4,29.4	36.3,39.9	28.5,30.7	38.7,42.9	30.8,33.3
$\sigma_D = 0.5$	-22.8,7.7	-40.0,-5.9	-20.1,9.9	-37.4,-3.9	-14.9,14.3	-32.4,0.0

jealousy-type relativity with $\Delta_i(\infty) \leq 0$, the AMTRs under Stackelberg equilibrium are in general larger than those in K&T (see Figure 7). However, if $\Delta_i(\infty) \geq 0$, the AMTRs under Stackelberg equilibrium are smaller than those in K&T for $\alpha_i(\infty)$ smaller than some critical value (see Figure 6). Under altruism-type relativity with $\Delta_i(\infty) \geq 0$, the AMTRs under Stackelberg equilibrium are always greater than those in K&T. However, if $\Delta_i(\infty) \leq 0$, the AMTRs under Stackelberg equilibrium are always smaller than those in K&T.

5.2 Nash vs. Stackelberg: the Leader Country

Tables 11-12 illustrate Proposition 4.2 by comparing AMTRs under these two types of equilibrium. As is obvious, no matter $\Delta_i(\infty) \geq 0$ or $\Delta_i(\infty) \leq 0$, the AMTRs in Nash equilibrium are in general smaller than those in Stackelberg equilibrium (see also Figures 8-9).

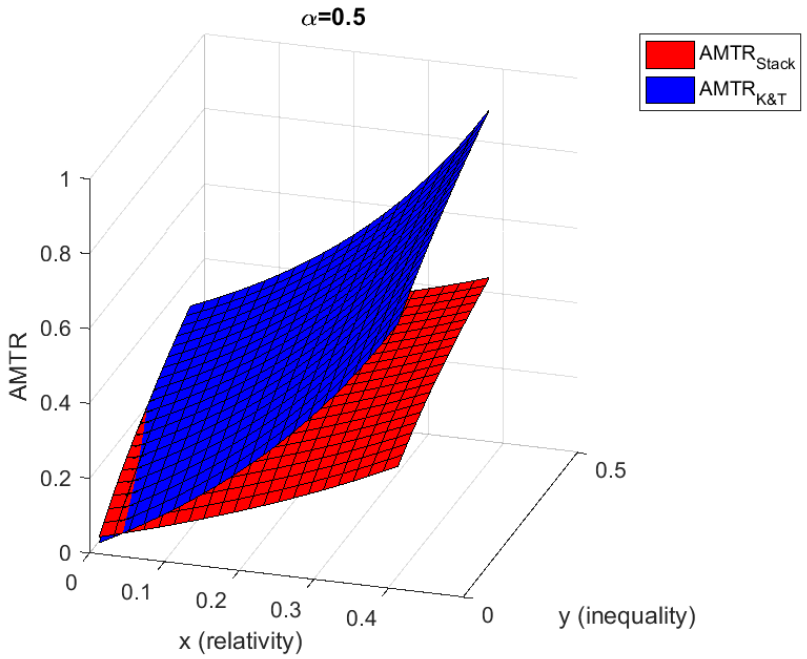

Figure 6: Parameters: $\alpha_i(\infty) = 0.5$, $\varepsilon = \tilde{\theta}_i(\infty) = 0.25$, $a_i > 1$, $\sigma_D < 0$, and $\sigma_F^2 = 0.04$.

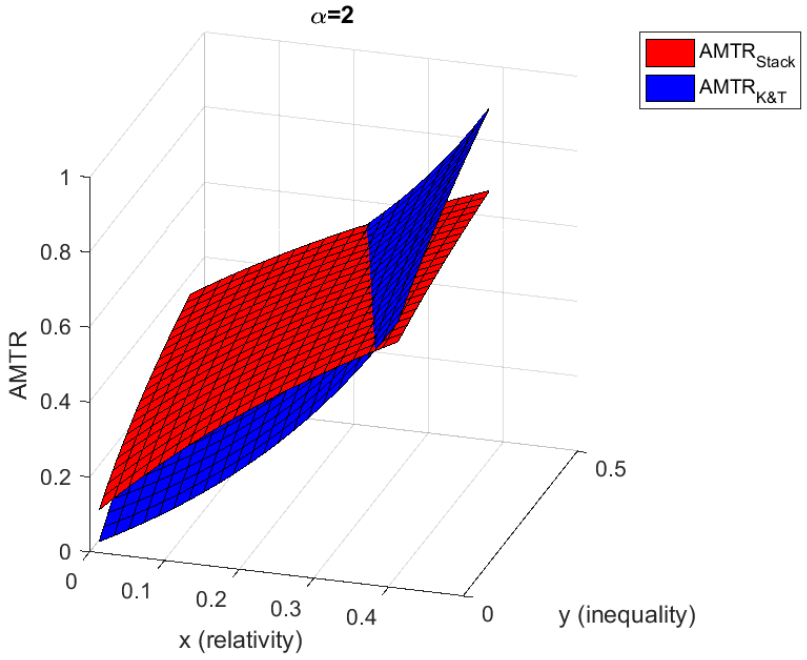

Figure 7: Parameters: $\alpha_i(\infty) = 2$, $\varepsilon = \tilde{\theta}_i(\infty) = 0.25$, $a_i > 1$, $\sigma_D < 0$, and $\sigma_F^2 = 0.04$.

6 Conclusion

In this paper, we develop a theoretical framework to analyze how the interplay of relative consumption concern and income inequality determines optimal income taxes in an international

Table 7: AMTR (%) under $\Delta_i(\infty) \geq 0$ with $\alpha_i(\infty) = 0.95$

	$\varepsilon = 0.25$	$\varepsilon = 0.25$	$\varepsilon = 0.33$	$\varepsilon = 0.33$	$\varepsilon = 0.5$	$\varepsilon = 0.5$
	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$
$\sigma_D = -0.5$	70.5,69.2	67.3,64.7	71.0,70.0	67.8,65.4	71.9,71.4	68.7,66.7
$\sigma_D = 0$	37.4,38.5	30.0,29.4	38.5,39.9	31.1,30.7	40.8,42.9	33.2,33.3
$\sigma_D = 0.5$	10.0,7.7	-1.4,-5.9	11.9,9.9	0.3,-3.9	15.4,14.3	3.6,0.0

Table 8: AMTR (%) under $\Delta_i(\infty) \geq 0$ with $\alpha_i(\infty) = 0.55$

	$\varepsilon = 0.25$	$\varepsilon = 0.25$	$\varepsilon = 0.33$	$\varepsilon = 0.33$	$\varepsilon = 0.5$	$\varepsilon = 0.5$
	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$
$\sigma_D = -0.5$	61.7,69.2	57.5,64.7	62.3,70.0	58.1,65.4	63.6,71.4	59.3,66.7
$\sigma_D = 0$	36.4,38.5	28.9,29.4	38.5,39.9	31.1,30.7	40.8,42.9	33.2,33.3
$\sigma_D = 0.5$	22.6,7.7	13.1,-5.9	24.1,9.9	14.5,-3.9	27.1,14.3	17.3,0.0

Table 9: AMTR (%) under $\Delta_i(\infty) \leq 0$ with $\alpha_i(\infty) = 1.05$

	$\varepsilon = 0.25$	$\varepsilon = 0.25$	$\varepsilon = 0.33$	$\varepsilon = 0.33$	$\varepsilon = 0.5$	$\varepsilon = 0.5$
	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$
$\sigma_D = -0.5$	72.1,69.2	69.1,64.7	72.5,70.0	69.5,65.4	73.5,71.4	70.4,66.7
$\sigma_D = 0$	37.6,38.5	30.2,29.4	38.8,39.9	31.3,30.7	41.0,42.9	33.5,33.3
$\sigma_D = 0.5$	6.2,7.7	-5.9,-5.9	8.1,9.9	-4.1,-3.9	11.9,14.3	-0.5,0.0

Table 10: AMTR (%) under $\Delta_i(\infty) \leq 0$ with $\alpha_i(\infty) = 1.55$

	$\varepsilon = 0.25$	$\varepsilon = 0.25$	$\varepsilon = 0.33$	$\varepsilon = 0.33$	$\varepsilon = 0.5$	$\varepsilon = 0.5$
	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$
$\sigma_D = -0.5$	78.0,69.2	75.7,64.7	78.4,70.0	76.1,65.4	79.1,71.4	76.7,66.7
$\sigma_D = 0$	38.7,38.5	31.5,29.4	38.8,39.9	31.3,30.7	41.0,42.9	33.5,33.3
$\sigma_D = 0.5$	-18.9,7.7	-35.4,-5.9	-16.4,9.9	-33.0,-3.9	-11.4,14.3	-28.1,0.0

setting with two competing countries. We establish and qualitatively characterize nonlinear labor income tax schedules that competing Rawlsian governments should implement when workers having private information on skills and migration costs decide where to live and how much to work. In addition to the case where governments play Nash, we also examine the scenario where they play Stackelberg.

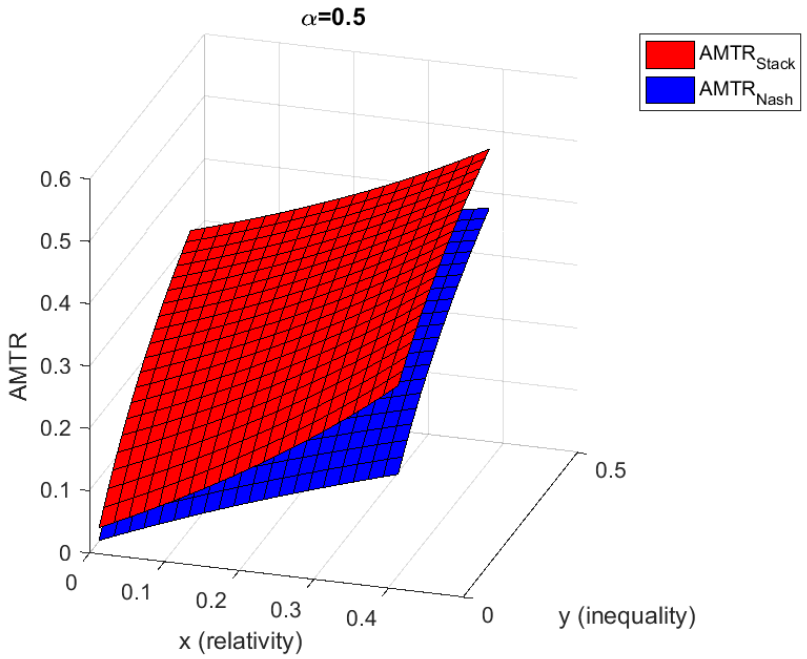

Figure 8: Parameters: $\alpha_i(\infty) = 0.5$, $\varepsilon = \tilde{\theta}_i(\infty) = 0.25$, $a_i > 1$, $\sigma_D < 0$, and $\sigma_F^2 = 0.04$.

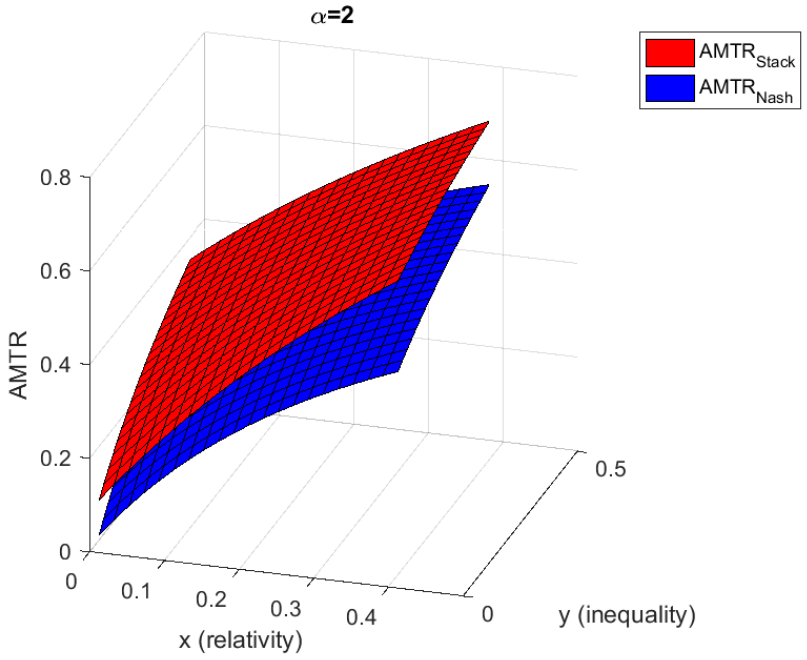

Figure 9: Parameters: $\alpha_i(\infty) = 2$, $\varepsilon = \tilde{\theta}_i(\infty) = 0.25$, $a_i > 1$, $\sigma_D < 0$, and $\sigma_F^2 = 0.04$.

First, we obtain an optimal tax formula that can be interpreted as an extension of those obtained by Diamond (1998), Saez (2001), Kanbur and Tuomala (2013), Lehmann et al. (2014) and Aronsson and Johansson-Stenman (2015). Second, we numerically calculate optimal AMTRs

Table 11: AMTR (%) under $\Delta_i(\infty) \geq 0$ with $\alpha_i(\infty) = 0.95$

	$\varepsilon = 0.25$	$\varepsilon = 0.25$	$\varepsilon = 0.33$	$\varepsilon = 0.33$	$\varepsilon = 0.5$	$\varepsilon = 0.5$
	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$
$\sigma_D = -0.5$	65.2,70.5	61.5,67.3	65.8,71.0	62.0,67.8	67.0,71.9	63.1,68.7
$\sigma_D = 0$	35.1,37.4	27.4,30.0	36.3,38.5	28.5,31.1	38.7,40.8	30.8,33.2
$\sigma_D = 0.5$	8.7,10.0	-3.0,-1.4	10.6,11.9	-1.3,0.3	14.2,15.4	2.2,3.6

Table 12: AMTR (%) under $\Delta_i(\infty) \leq 0$ with $\alpha_i(\infty) = 1.05$

	$\varepsilon = 0.25$	$\varepsilon = 0.25$	$\varepsilon = 0.33$	$\varepsilon = 0.33$	$\varepsilon = 0.5$	$\varepsilon = 0.5$
	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$	$a_i = 2$	$a_i = 3$
$\sigma_D = -0.5$	66.8,72.1	63.3,69.1	67.4,72.5	63.8,69.5	68.5,73.5	64.8,70.4
$\sigma_D = 0$	35.1,37.6	27.4,30.2	36.3,38.8	28.5,31.3	38.7,41.0	30.8,33.5
$\sigma_D = 0.5$	4.6,6.2	-7.7,-5.9	6.6,8.1	-5.9,-4.1	10.4,11.9	-2.2,-0.5

under both types of equilibrium and compare them to those obtained using the formula of K&T, finding that the country with large labor inflow imposes a much smaller marginal tax rate and the country with large labor outflow imposes a much higher marginal tax rate than suggested by K&T. This finding holds for sufficiently various combinations of relative consumption and income inequality. Third, for the case with quasi-linear in consumption preferences and jealousy-type consumption comparison, we show that the leader country imposes a higher marginal tax rate in Stackelberg equilibrium than that it may impose in Nash equilibrium. And fourth, we provide a complete characterization on how relativity and inequality together determine the optimal AMTR under both Nash and Stackelberg tax competition, finding that both the elasticity and level of migration play determinant roles in determining whether they are complementary or substitutive in shaping the optimal tax rates placed on top-income workers. We, therefore, demonstrate that the optimal redistributive taxation policy for countries involved in globalization should not ignore these important effects resulted from tax-driven migrations as well as the interplay of relativity and inequality. Since alternative forms of tax competition generate heterogenous effects on optimal tax rates, the identification of the form of tax competition should be of practical relevance.

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Appendix: Proofs

Proof of Theorem 3.1. We shall complete the proof in 3 steps.

Step 1. Given the FOC (4) of individual choice, the indirect utility of a type-w worker in country $i \in \{A, B\}$ can be written as:

$$U_i(w) = v(\varphi_i(U_i(w), l_i(w), \mu_i, \mu_{-i})) - h(l_i(w)) + \psi(\mu_i, \mu_{-i}), \tag{15}$$

where we treat individual consumption $c_i(w)$ as an implicit function of $U_i(w)$, $l_i(w)$, μ_i, μ_{-i}, and equivalently rewrite it as $\varphi_i(\cdot)$. By applying the Implicit Function Theorem, we get from (15) that:

$$\frac{\partial \varphi_i}{\partial l_i} = \frac{h'(l_i(w))}{v'(c_i(w))}, \ \frac{\partial \varphi_i}{\partial U_i} = \frac{1}{v'(c_i(w))}, \ \frac{\partial \varphi_i}{\partial \mu_i} = -\frac{\psi_i(\mu_i, \mu_{-i})}{v'(c_i(w))} \ \text{ and } \ \frac{\partial \varphi_i}{\partial \mu_{-i}} = -\frac{\psi_{-i}(\mu_i, \mu_{-i})}{v'(c_i(w))}. \tag{16}$$

Step 2. For expositional purposes, we follow the first-order approach and ignore the SOIC conditions. After deriving the solutions, then we can verify whether the SOIC conditions are binding or not. The corresponding Lagrangian is:

$$\begin{aligned}
&\mathcal{L}_i(\{U_i(w), l_i(w)\}_{w\in[\underline{w},\overline{w}]}, \mu_i; \lambda_i, \gamma_i, \{\varsigma_i(w)\}_{w\in[\underline{w},\overline{w}]}) \\
= \ & U_i(\underline{w}) + \lambda_i \int_{\underline{w}}^{\overline{w}} \left\{ [w l_i(w) - \varphi_i(U_i(w), l_i(w), \mu_i, \mu_{-i})] \phi_i(U_i(w) - U_{-i}(w); w) - \frac{R}{\overline{w} - \underline{w}} \right\} dw \\
& + \int_{\underline{w}}^{\overline{w}} \varsigma_i(w) \left[h'(l_i(w)) \frac{l_i(w)}{w} - \dot{U}_i(w) \right] dw \\
& + \gamma_i \left[\mu_i - \int_{\underline{w}}^{\overline{w}} \varphi_i(U_i(w), l_i(w), \mu_i, \mu_{-i}) f_i(w) dw \right]
\end{aligned} \tag{17}$$

where $\lambda_i > 0$ is the multiplier associated with the binding budget constraint (9), $\varsigma_i(w)$ is the multiplier associated with the FOIC conditions (11), and γ_i is the multiplier associated with the comparison consumption constraint (2). Integrating by parts, we obtain

$$\int_{\underline{w}}^{\overline{w}} \varsigma_i(w) \dot{U}_i(w) dw = \varsigma_i(\overline{w}) U_i(\overline{w}) - \varsigma_i(\underline{w}) U_i(\underline{w}) - \int_{\underline{w}}^{\overline{w}} \dot{\varsigma}_i(w) U_i(w) dw. \tag{18}$$

Plugging (18) in (17) gives rise to:

$$\begin{aligned}
&\mathcal{L}_i(\{U_i(w), l_i(w)\}_{w\in[\underline{w},\overline{w}]}, \mu_i; \lambda_i, \gamma_i, \{\varsigma_i(w)\}_{w\in[\underline{w},\overline{w}]}) \\
= \ & U_i(\underline{w}) + \lambda_i \int_{\underline{w}}^{\overline{w}} \left\{ [w l_i(w) - \varphi_i(U_i(w), l_i(w), \mu_i, \mu_{-i})] \phi_i(U_i(w) - U_{-i}(w); w) - \frac{R}{\overline{w} - \underline{w}} \right\} dw \\
& + \varsigma_i(\underline{w}) U_i(\underline{w}) - \varsigma_i(\overline{w}) U_i(\overline{w}) + \int_{\underline{w}}^{\overline{w}} \left[\varsigma_i(w) h'(l_i(w)) \frac{l_i(w)}{w} + \dot{\varsigma}_i(w) U_i(w) \right] dw \\
& + \gamma_i \left[\mu_i - \int_{\underline{w}}^{\overline{w}} \varphi_i(U_i(w), l_i(w), \mu_i, \mu_{-i}) f_i(w) dw \right].
\end{aligned}$$

Assuming that there is no bunching of workers of different skills and the existence of an interior solution, applying (16) shows that the necessary conditions can be written as:

$$\begin{aligned}\frac{\partial \mathcal{L}_i}{\partial l_i(w)} = \lambda_i \left[w - \frac{h'(l_i(w))}{v'(c_i(w))}\right] \tilde{f}_i(w) - \gamma_i \frac{h'(l_i(w))}{v'(c_i(w))} f_i(w) \\ + \frac{\varsigma_i(w) h'(l_i(w))}{w}\left[1 + \frac{l_i(w) h''(l_i(w))}{h'(l_i(w))}\right] = 0 \quad \forall w \in [\underline{w}, \overline{w}],\end{aligned} \tag{19}$$

$$\begin{aligned}\frac{\partial \mathcal{L}_i}{\partial U_i(w)} = -\frac{\lambda_i \tilde{f}_i(w)}{v'(c_i(w))} + \lambda_i T_i(y_i(w)) \tilde{\eta}_i(w) \tilde{f}_i(w) \\ - \frac{\gamma_i f_i(w)}{v'(c_i(w))} + \dot{\varsigma}_i(w) = 0 \quad \forall w \in (\underline{w}, \overline{w}),\end{aligned} \tag{20}$$

$$\frac{\partial \mathcal{L}_i}{\partial U_i(\underline{w})} = 1 + \varsigma_i(\underline{w}) = 0, \tag{21}$$

$$\frac{\partial \mathcal{L}_i}{\partial U_i(\overline{w})} = -\varsigma_i(\overline{w}) = 0, \tag{22}$$

$$\frac{\partial \mathcal{L}_i}{\partial \mu_i} = \lambda_i \int_{\underline{w}}^{\overline{w}} \frac{\psi_i(\mu_i, \mu_{-i})}{v'(c_i(w))} \tilde{f}_i(w) dw + \gamma_i \left[1 + \int_{\underline{w}}^{\overline{w}} \frac{\psi_i(\mu_i, \mu_{-i})}{v'(c_i(w))} f_i(w) dw\right] = 0. \tag{23}$$

Using (20), we get:

$$\frac{\dot{\varsigma}_i(w)}{\lambda_i} = \frac{\gamma_i}{\lambda_i} \frac{f_i(w)}{v'(c_i(w))} + \frac{\tilde{f}_i(w)}{v'(c_i(w))} - T_i(y_i(w)) \tilde{\eta}_i(w) \tilde{f}_i(w).$$

Integrating on both sides of this equation and using the transversality condition (22), we obtain:

$$-\frac{\varsigma_i(w)}{\lambda_i} = \frac{\gamma_i}{\lambda_i} \int_w^{\overline{w}} \frac{f_i(t)}{v'(c_i(t))} dt + \int_w^{\overline{w}} \left[\frac{1}{v'(c_i(t))} - T_i(y_i(t)) \tilde{\eta}_i(t)\right] \tilde{f}_i(t) dt. \tag{24}$$

Rearranging (19) via using FOC (4), we have:

$$\frac{T_i'(y_i(w))}{1 - T_i'(y_i(w))} = \frac{\gamma_i}{\lambda_i} \frac{f_i(w)}{\tilde{f}_i(w)} - \frac{\varsigma_i(w)}{\lambda_i} \frac{v'(c_i(w))}{w \tilde{f}_i(w)} \left[1 + \frac{l_i(w) h''(l_i(w))}{h'(l_i(w))}\right] \quad \forall w \in [\underline{w}, \overline{w}]. \tag{25}$$

Substituting (24) into (25) gives the first-order conditions characterizing the optimum marginal tax rates, with γ_i/λ_i determined by solving (23).

Step 3. To derive a sufficient condition for the optimal marginal tax profile to satisfy the SOIC conditions, we rewrite the FOC (4) as

$$\underbrace{\frac{v'(c_i(w))}{h'(y_i(w)/w)}}_{\text{LHS}} = \underbrace{\frac{1}{w[1 - T_i'(y_i(w))]}}_{\text{RHS}}. \tag{26}$$

Noting that

$$\frac{\mathrm{dLHS}}{\mathrm{d}w} = \frac{v''(c_i(w)) \dot{c}_i(w)}{h'(y_i(w)/w)} - \frac{v'(c_i(w)) h''(y_i(w)/w)[w \dot{y}_i(w) - y_i(w)]}{[w h'(y_i(w)/w)]^2}$$

and $c_i(w) = y_i(w) - T_i(y_i(w)) \Rightarrow \dot{c}_i(w) = \dot{y}_i(w)[1 - T_i'(y_i(w))]$, thus:

$$\frac{\mathrm{dLHS}}{\mathrm{d}w} < 0 \implies \dot{y}_i(w) > 0. \tag{27}$$

Also, noting that

$$\frac{\mathrm{dRHS}}{\mathrm{d}w} = -\left\{w\left[1-T_i'(y_i(w))\right]\right\}^2\left[1-T_i'(y_i(w))-w\frac{\mathrm{d}T_i'(y_i(w))}{\mathrm{d}w}\right],$$

thus:

$$\frac{\mathrm{d}T_i'(y_i(w))}{\mathrm{d}w}\leq 0 \implies \frac{\mathrm{dRHS}}{\mathrm{d}w}<0. \tag{28}$$

Therefore, (26) combined with (27) and (28) implies that

$$\frac{\mathrm{d}T_i'(y_i(w))}{\mathrm{d}w}\leq 0 \implies \dot{y}_i(w)>0,$$

as desired. ■

Proof of Lemma 3.1. The Lagrangian of government i' problem can be expressed as:

$$\begin{aligned}
&\mathcal{L}_i(\{U_i(w),l_i(w)\}_{w\in[\underline{w},\overline{w}]},\mu_i;\lambda_i,\gamma_i)\\
=\;& U_i(\underline{w})+\lambda_i\int_{\underline{w}}^{\overline{w}}\left\{[wl_i(w)-\varphi_i(U_i(w),l_i(w),\mu_i,\mu_{-i})]\phi_i(U_i(w)-U_{-i}(w);w)-\frac{R}{\overline{w}-\underline{w}}\right\}dw\\
&+\gamma_i\left[\mu_i-\int_{\underline{w}}^{\overline{w}}\varphi_i(U_i(w),l_i(w),\mu_i,\mu_{-i})f_i(w)dw\right].
\end{aligned}$$

where $\lambda_i>0$ is the multiplier associated with the binding budget constraint (9) and γ_i is the multiplier associated with the comparison consumption constraint (2). Assuming that there is no bunching of workers of different skills and the existence of an interior solution, applying (16) gives these necessary conditions:

$$\frac{\partial\mathcal{L}_i}{\partial U_i(w)}=\lambda_i\left[T_i(y_i(w))\tilde{\eta}_i(w)-\frac{1}{v'(c_i(w))}\right]\tilde{f}_i(w)-\gamma_i\frac{f_i(w)}{v'(c_i(w))}=0\;\;\forall w\in(\underline{w},\overline{w}], \tag{29}$$

$$\frac{\partial\mathcal{L}_i}{\partial l_i(w)}=\lambda_i\left(w-\frac{h'(l_i(w))}{v'(c_i(w))}\right)\tilde{f}_i(w)-\gamma_i\frac{h'(l_i(w))f_i(w)}{v'(c_i(w))}=0\;\;\forall w\in[\underline{w},\overline{w}], \tag{30}$$

and

$$\frac{\partial\mathcal{L}_i}{\partial\mu_i}=\lambda_i\int_{\underline{w}}^{\overline{w}}\frac{\psi_i(\mu_i,\mu_{-i})}{v'(c_i(w))}\tilde{f}_i(w)dw+\gamma_i\left[1+\int_{\underline{w}}^{\overline{w}}\frac{\psi_i(\mu_i,\mu_{-i})}{v'(c_i(w))}f_i(w)dw\right]=0. \tag{31}$$

By using (29), we obtain the Tiebout-best tax liabilities. By using (30) and the FOC (4), we obtain the Tiebout-best marginal tax rates. The ratio γ_i/λ_i is determined by (31). As is obvious, the least productive workers receive a transfer determined by the government's budget constraint. Therefore, the optimal tax function is discontinuous at $w=\underline{w}$. ■

Proof of Proposition 3.1. We shall complete the proof in 6 steps.

Step 1. By Assumption 2.1, $\gamma_i/\lambda_i>0$ is guaranteed. Given that $v(\cdot)$ is strictly increasing and $h(\cdot)$ is strictly increasing and convex, for (i) to hold it suffices to show that $\mathcal{C}_i(w)\geq 0$ for

$\forall w \in (\underline{w}, \overline{w})$. Therefore, by directly comparing the formulas of marginal tax rates established in Theorem 3.1 and Lemma 3.1, claim (i) is immediate.

Step 2. By applying the transversality condition (21) to equation (25), it is easy to see that

$$\frac{T_i'(y_i(\underline{w}))}{1 - T_i'(y_i(\underline{w}))} > \frac{\gamma_i}{\lambda_i} \frac{f_i(\underline{w})}{\tilde{f}_i(\underline{w})} > 0$$

under Assumption 2.1. Similarly, for a bounded skill distribution with $\overline{w} < \infty$, applying the transversality condition (22) to equation (25) gives:

$$\frac{T_i'(y_i(\overline{w}))}{1 - T_i'(y_i(\overline{w}))} = \frac{\gamma_i}{\lambda_i} \frac{f_i(\overline{w})}{\tilde{f}_i(\overline{w})} > 0$$

under Assumption 2.1. By using Lemma 3.1 again, the required assertion (ii) follows.

Step 3. Suppose $h(\cdot)$ takes the isoelastic form, then $\mathcal{A}_i(w)$ is a positive constant. Suppose the first-order approach is valid, namely the SOIC conditions are not binding in the Nash equilibrium, then we have that $v'(\cdot)$ is strictly deceasing in w as $v(\cdot)$ is assumed to be strictly concave. With single-peaked skill distributions, $1/wf_i(w)$ always decreases before the mode w_m. Beyond the mode, it either increases or decreases, depending on how rapidly $f_i(w)$ falls with w. A sufficient, though not necessary, condition for decreasing $T_i'(\cdot)$ over the entire skill distribution is that aggregate skills $wf_i(w)$ are non-decreasing beyond the mode w_m. Also, noting from term $\mathcal{C}_i(w)$ that

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}w} \int_w^{\overline{w}} \left\{ \frac{1}{v'(c_i(t))} \left[1 + \frac{\gamma_i}{\lambda_i} \frac{f_i(t)}{\tilde{f}_i(t)} \right] - T_i(y_i(t)) \tilde{\eta}_i(t) \right\} \tilde{f}_i(t) dt \\ = &\left\{ T_i(y_i(w)) - \frac{1}{v'(c_i(w)) \tilde{\eta}_i(w)} \left[1 + \frac{\gamma_i}{\lambda_i} \frac{f_i(w)}{\tilde{f}_i(w)} \right] \right\} \tilde{\eta}_i(w) \tilde{f}_i(w), \end{aligned}$$

hence an application of Lemma 3.1 completes the proof of claim (iii).

Step 4. Define

$$\Sigma_i(w) \equiv -\frac{\varsigma_i(w)}{\lambda_i} = \frac{\gamma_i}{\lambda_i} \int_w^{\overline{w}} \frac{f_i(t)}{v'(c_i(t))} dt + \int_w^{\overline{w}} \left[\frac{1}{v'(c_i(t))} - T_i(y_i(t)) \tilde{\eta}_i(t) \right] \tilde{f}_i(t) dt,$$

then the signs of $\Sigma_i(w)$ and $\varsigma_i(w)$ are opposite. As $\varsigma_i(w)$ is differentiable, $\Sigma_i(w)$ is differentiable as well and we have:

$$\Sigma_i'(w) = \left\{ T_i(y_i(w)) - \frac{1}{v'(c_i(w)) \tilde{\eta}_i(w)} \left[1 + \frac{\gamma_i}{\lambda_i} \frac{f_i(w)}{\tilde{f}_i(w)} \right] \right\} \tilde{\eta}_i(w) \tilde{f}_i(w) \equiv \xi_i(w) \tilde{\eta}_i(w) \tilde{f}_i(w), \quad (32)$$

which implies that $\Sigma_i'(w)$ and $\xi_i(w)$ have the same sign. Note that

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}w}\left\{\frac{1}{v'(c_i(w)\tilde{\eta}_i(w))}\left[1+\frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}\right]\right\}\\
=&\underbrace{-\frac{v''(c_i(w))\dot{c}_i(w)}{[v'(c_i(w))]^2\tilde{\eta}_i(w)}\left[1+\frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}\right]}_{>0}\\
&-\frac{\dot{\tilde{\eta}}_i(w)}{v'(c_i(w))[\tilde{\eta}_i(w)]^2}\left[1+\frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}\right]+\underbrace{\frac{1}{v'(c_i(w))\tilde{\eta}_i(w)}\frac{\gamma_i}{\lambda_i}\frac{\mathrm{d}[f_i(w)/\tilde{f}_i(w)]}{\mathrm{d}w}}_{\leq 0}
\end{aligned}$$

under Assumption 2.1 and the assumptions that $\dot{y}_i(w)>0$ and $f_i(w)/\tilde{f}_i(w)$ is non-increasing in w, thus the condition $-\frac{v''(c_i(w))\dot{c}_i(w)}{v'(c_i(w))}\leq\frac{\dot{\tilde{\eta}}_i(w)}{\tilde{\eta}_i(w)}$ is sufficient for

$$\frac{\mathrm{d}}{\mathrm{d}w}\left\{\frac{1}{v'(c_i(w)\tilde{\eta}_i(w))}\left[1+\frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}\right]\right\}\leq 0. \tag{33}$$

If we assume that $\Sigma_i(w)\geq 0$, then we get from the optimal tax formula in Theorem 3.1 that $T_i'(y_i(w))>0$. Then applying (32) and (33) shows that $\xi_i'(w)>0$ given $\Sigma_i(w)\geq 0$.

Step 5. Assume that there exists a $\tilde{w}\in(\underline{w},\overline{w})$ such that $\Sigma_i(\tilde{w})\geq 0$. Then we have two cases to consider in what follows, namely either $\Sigma_i'(\tilde{w})\geq 0$ or $\Sigma_i'(\tilde{w})<0$. If $\Sigma_i'(\tilde{w})\geq 0$, then we have both $\xi_i(\tilde{w})\geq 0$ and $\xi_i'(\tilde{w})>0$. So the continuity of $\xi_i(w)$ with respect to w implies that there is an open interval with lower bound $\tilde{w}$ such that $\xi_i(\cdot)>0$, and hence $\Sigma_i'(\cdot)>0$, on this interval. $\Sigma_i(\cdot)$ is thus positive and strictly increasing on this interval. Without loss of generality, let $(\tilde{w},\hat{w})$ be a maximal interval on which $\Sigma_i'(w)>0$ with $\tilde{w}<\hat{w}\leq\overline{w}$. As a consequence, $0\leq\Sigma_i(\tilde{w})<\Sigma(\hat{w})$, which implies that $\xi_i'(w)>0$ for $\forall w\in[\tilde{w},\hat{w}]$. As a result, $0\leq\xi_i(\tilde{w})<\xi_i(\hat{w})$, which leads us to $\Sigma_i'(\hat{w})>0$ by using (32). Therefore, $\Sigma_i(\cdot)$ is increasing on $[\tilde{w},\overline{w}]$ given that $\Sigma_i'(\tilde{w})\geq 0$. We know from the transversality condition (22) that $\Sigma_i(\overline{w})=-\varsigma_i(\overline{w})/\lambda_i=0$. As we have already shown that $0\leq\Sigma_i(\tilde{w})<\Sigma_i(\overline{w})$, an immediate contradiction occurs. We, accordingly, claim that $\Sigma_i'(\tilde{w})\geq 0$ does not hold true.

Step 6. Given that we have shown that $\Sigma_i'(\tilde{w})<0$ for the chosen $\tilde{w}$, we thus have $\xi_i(\tilde{w})<0$ by (32) and $\xi_i'(\tilde{w})>0$. Similarly, the continuity of $\xi_i(w)$ with respect to w implies that there is an open interval with upper bound $\tilde{w}$ such that $\xi_i(\cdot)<0$, and hence $\Sigma_i'(\cdot)<0$, on this interval. $\Sigma_i(\cdot)$ is thus positive and strictly decreasing on this interval. Without loss of generality, let $(w^*,\tilde{w})$ be a maximal interval on which $\Sigma_i'(w)<0$ with $\underline{w}\leq w^*<\tilde{w}$. In consequence, $0\leq\Sigma_i(\tilde{w})<\Sigma(w^*)$, which implies that $\xi_i'(w)>0$ for $\forall w\in[w^*,\tilde{w}]$. As a result, $0>\xi_i(\tilde{w})>\xi_i(w^*)$, which leads us to $\Sigma_i'(w^*)<0$ by using (32). Therefore, $\Sigma_i(\cdot)$ will not stop decreasing until reaching the lower bound $\underline{w}$, namely $\Sigma_i(\cdot)$ will be decreasing on $[\underline{w},\tilde{w}]$. We know from the transversality condition (21) that $\Sigma_i(\underline{w})=-\varsigma_i(\underline{w})/\lambda_i>0$. Since we have already shown that $0\leq\Sigma_i(\tilde{w})<\Sigma_i(\underline{w})$, thus

the transversality condition is fulfilled in this case. By using (32) and Lemma 3.1 again, the required assertion (iv) follows. ■

Proof of Proposition 3.2. We shall complete the proof in 5 steps.

Step 1. It is easy to verify that $\gamma_i/\lambda_i < 0$ under Assumption 2.2. Thus, comparing the optimal tax formulas in Theorem 3.1 and Lemma 3.1, it is immediate that $T_i'(y_i(\overline{w})) = T_i^{*'}(y_i(\overline{w})) < 0$ for $\overline{w} < \infty$. Noting from (21) that $\varsigma_i(\underline{w}) = -1$, thus $\mathcal{A}_i(\underline{w})\mathcal{B}_i(\underline{w})\mathcal{C}_i(\underline{w}) > 0$, which implies that $T_i'(y_i(\underline{w})) > T_i^{*'}(y_i(\underline{w}))$ by using Lemma 3.1 again. The proof of claim (i) is hence complete.

Step 2. Note that

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}w}\left\{\frac{1}{v'(c_i(w)\tilde{\eta}_i(w))}\left[1+\frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}\right]\right\} \\
&= \underbrace{-\frac{v''(c_i(w))\dot{c}_i(w)}{[v'(c_i(w))]^2\tilde{\eta}_i(w)}\left[1+\frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}\right]}_{\geq 0 \text{ given } \Gamma_i(w)\geq 0} \\
&\quad \underbrace{-\frac{\dot{\tilde{\eta}}_i(w)}{v'(c_i(w))[\tilde{\eta}_i(w)]^2}\left[1+\frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}\right]+\frac{1}{v'(c_i(w))\tilde{\eta}_i(w)}\frac{\gamma_i}{\lambda_i}\frac{\mathrm{d}[f_i(w)/\tilde{f}_i(w)]}{\mathrm{d}w}}_{\equiv\, \Xi_i(w)},
\end{aligned} \tag{34}$$

we get by rearranging the terms that

$$\Xi_i(w) \geq 0 \iff \dot{\Gamma}_i(w) \geq \Gamma_i(w)\frac{\dot{\tilde{\eta}}_i(w)}{\tilde{\eta}_i(w)}. \tag{35}$$

It follows from (32) that

$$\xi_i'(w) = T_i'(y_i(w))\dot{y}_i(w) - \frac{\mathrm{d}}{\mathrm{d}w}\left\{\frac{1}{v'(c_i(w)\tilde{\eta}_i(w))}\left[1+\frac{\gamma_i}{\lambda_i}\frac{f_i(w)}{\tilde{f}_i(w)}\right]\right\}. \tag{36}$$

We now show that, under Assumption 2.2, $\Sigma_i(w) \leq 0$ implies $\xi_i'(w) < 0$, with $\Sigma_i(w)$ defined in the proof of Proposition 3.1. Assume $\Sigma_i(w) \leq 0$, then we get from the optimal tax formula established in Theorem 3.1 that $T_i'(y_i(w))\dot{y}_i(w) < 0$ under Assumption 2.2. Therefore, it follows from (34)-(36) and the assumption $\dot{y}_i(w) > 0$ under the first-order approach that $\xi_i'(w) < 0$, as desired.

Step 3. Here we prove this claim: if there exists a $\tilde{w} \in [\underline{w}, \overline{w})$ such that $\Sigma_i(\tilde{w}) \leq 0$ and $\Sigma_i'(\tilde{w}) \leq 0$, then $\Sigma_i(\cdot)$ is decreasing on the closed interval $[\tilde{w}, \overline{w}]$. Given these assumptions, then we have both $\xi_i(\tilde{w}) \leq 0$ and $\xi_i'(\tilde{w}) < 0$. So the continuity of $\xi_i(w)$ with respect to w implies that there is an open interval with lower bound $\tilde{w}$ such that $\xi_i(\cdot) < 0$, and hence $\Sigma_i'(\cdot) < 0$, on this interval. $\Sigma_i(\cdot)$ is thus negative and strictly decreasing on this interval. Without loss of generality, let $(\tilde{w}, \hat{w})$ be a maximal interval on which $\Sigma_i'(w) < 0$ with $\tilde{w} < \hat{w} \leq \overline{w}$. As a consequence, $\Sigma(\hat{w}) < \Sigma_i(\tilde{w}) \leq 0$, which implies that $\xi_i'(w) < 0$ for $\forall w \in [\tilde{w}, \hat{w}]$. Consequently,

$\xi_i(\hat{w}) < \xi_i(\tilde{w}) \leq 0$, which leads us to $\Sigma_i'(\hat{w}) < 0$ by using (32). Therefore, $\Sigma_i(\cdot)$ is decreasing on $[\tilde{w}, \overline{w}]$, as desired.

Step 4. Here we prove another claim: if there exists a $\tilde{w} \in (\underline{w}, \overline{w}]$ such that $\Sigma_i(\tilde{w}) \leq 0$ and $\Sigma_i'(\tilde{w}) \geq 0$, then $\Sigma_i(\cdot)$ is increasing on the closed interval $[\underline{w}, \tilde{w}]$. Given these assumptions, we thus have $\xi_i(\tilde{w}) \geq 0$ by (32) and $\xi_i'(\tilde{w}) < 0$. Similarly, the continuity of $\xi_i(w)$ with respect to w implies that there is an open interval with upper bound $\tilde{w}$ such that $\xi_i(\cdot) > 0$, and hence $\Sigma_i'(\cdot) > 0$, on this interval. $\Sigma_i(\cdot)$ is thus negative and strictly increasing on this interval. Without loss of generality, let $(w^*, \tilde{w})$ be a maximal interval on which $\Sigma_i'(w) > 0$ with $\underline{w} \leq w^* < \tilde{w}$. In consequence, $\Sigma(w^*) < \Sigma_i(\tilde{w}) \leq 0$, which implies that $\xi_i'(w) < 0$ for $\forall w \in [w^*, \tilde{w}]$. Consequently, $0 \leq \xi_i(\tilde{w}) < \xi_i(w^*)$, which leads us to $\Sigma_i'(w^*) > 0$ by using (32). Therefore, $\Sigma_i(\cdot)$ will not stop increasing until reaching the lower bound $\underline{w}$, namely $\Sigma_i(\cdot)$ will be increasing on $[\underline{w}, \tilde{w}]$, as desired.

Step 5. We now show that $\Sigma_i(w) > 0$, and hence $\mathcal{A}_i(w)\mathcal{B}_i(w)\mathcal{C}_i(w) > 0$, on $(\underline{w}, \overline{w})$. In fact, we prove this result by means of contradiction. It follows from the transversality condition (22) that $\Sigma_i(\overline{w}) = -\varsigma_i(\overline{w})/\lambda_i = 0$. If there exists a $\tilde{w} \in [\underline{w}, \overline{w})$ such that $\Sigma_i(\tilde{w}) \leq 0$ and $\Sigma_i'(\tilde{w}) \leq 0$, then we have proven in Step 3 that $\Sigma_i(\cdot)$ is decreasing on the closed interval $[\tilde{w}, \overline{w}]$. This means that $\Sigma_i(\overline{w}) < \Sigma_i(\tilde{w}) \leq 0$, yielding an immediate contradiction. Also, it follows from the transversality condition (21) that $\Sigma_i(\underline{w}) = -\varsigma_i(\underline{w})/\lambda_i > 0$. If there exists a $\tilde{w} \in (\underline{w}, \overline{w}]$ such that $\Sigma_i(\tilde{w}) \leq 0$ and $\Sigma_i'(\tilde{w}) \geq 0$, then we have proven in Step 4 that $\Sigma_i(\cdot)$ is increasing on the closed interval $[\underline{w}, \tilde{w}]$. This means that $\Sigma_i(\underline{w}) < \Sigma_i(\tilde{w}) \leq 0$, yielding an immediate contradiction. We can thus conclude that $\Sigma_i(w) > 0$ for $\forall w \in (\underline{w}, \overline{w})$ and also $\Sigma_i'(\underline{w}) > 0 > \Sigma_i'(\overline{w})$. By using Lemma 3.1 and (32) again, then the proof of claim (ii) is complete. ■

Proof of Proposition 3.3. It follows from condition (b) that $\mathcal{A}_i(w) = 1 + \varepsilon$, a fixed positive constant. The ex post skill distribution term $\mathcal{B}_i(w)$ can be decomposed through:

$$\mathcal{B}_i(w) = \frac{1 - F_i(w)}{w f_i(w)} \cdot \frac{\frac{\tilde{F}_i(\infty) - \tilde{F}_i(w)}{1 - F_i(w)}}{\tilde{f}_i(w)/f_i(w)}.$$

By condition (c), we have $\frac{1-F_i(w)}{w f_i(w)} = 1/a_i$. By L'Hôpital's rule, we obtain

$$\lim_{w \uparrow \infty} \frac{\tilde{F}_i(\infty) - \tilde{F}_i(w)}{1 - F_i(w)} = \lim_{w \uparrow \infty} \frac{\tilde{f}_i(w)}{f_i(w)}.$$

As a result, $\lim_{w \uparrow \infty} \mathcal{B}_i(w) = 1/a_i$. By using the definition of the elasticity of migration and conditions (a) and (c), term $\mathcal{C}_i(w)$ can be rewritten as:

$$\mathcal{C}_i(w) = \frac{\int_w^\infty \left[1 + \frac{\gamma_i}{\lambda_i} \frac{f_i(t)}{\tilde{f}_i(t)} - \frac{T_i(y_i(t))}{y_i(t) - T_i(y_i(t))} \tilde{\theta}_i(t)\right] \tilde{f}_i(t) dt}{\tilde{F}_i(\infty) - \tilde{F}_i(w)}.$$

Thus, making use of the L'Hôpital's rule again shows that

$$\lim_{w\uparrow\infty} \mathcal{C}_i(w) = 1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) - \frac{T_i'(y_i(\infty))}{1 - T_i'(y_i(\infty))}\tilde{\theta}_i(\infty).$$

So, we get from the optimal tax formula derived in Theorem 3.1 that

$$\frac{T_i'(y_i(\infty))}{1 - T_i'(y_i(\infty))} = \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + (1+\varepsilon)\frac{1}{a_i}\left[1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) - \frac{T_i'(y_i(\infty))}{1 - T_i'(y_i(\infty))}\tilde{\theta}_i(\infty)\right],$$

rearranging the algebra of which gives the desired optimal asymptotic tax rate. ■

Proof of Proposition 3.4. We shall complete the proof in 3 steps.

Step 1. By applying condition (a) assumed in Proposition 3.3 and the assumption $\psi_i(\mu_i, \mu_{-i}) = \sigma_D \in (-1, 0)$ to equation (14) produces:

$$\frac{\gamma_i}{\lambda_i} = \frac{-\sigma_D}{1+\sigma_D}\tilde{F}_i(\infty) > 0,$$

in which it is unnecessary that $\tilde{F}_i(\infty) = 1$. Also, if $F_i(w) = F_{-i}(w)$, then by using the definition of $\tilde{f}_i(w)$ we obtain $\partial\alpha_i(\infty)/\partial(1/a_i) = 0$. Therefore, as long as $\partial\tilde{F}_i(\infty)/\partial(1/a_i) = 0$, we must have $\partial\left(\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)\right)/\partial(1/a_i) = 0$. In addition, it follows from the definition of $\tilde{\theta}_i(w)$ that $\partial\tilde{\theta}_i(\infty)/\partial(1/a_i) = 0$. Finally, it is straightforward that $\partial(\gamma_i/\lambda_i)/\partial(-\sigma_D) > 0$.

Step 2. Using the established formula of $T_i'(y_i(\infty))$, we have:

$$\frac{\partial T_i'(y_i(\infty))}{\partial\left(\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)\right)} = \frac{[1 + (1+\varepsilon)(1/a_i)]\left[1 + (1+\varepsilon)(1/a_i)\tilde{\theta}_i(\infty)\right]}{\left\{1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + (1+\varepsilon)(1/a_i)\left[1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + \tilde{\theta}_i(\infty)\right]\right\}^2},$$

by which we hence obtain:

$$\begin{aligned}
&\frac{\partial^2 T_i'(y_i(\infty))}{\partial\left(\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)\right)\partial(1/a_i)} \\
&= \frac{1+\varepsilon}{\left\{1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + (1+\varepsilon)(1/a_i)\left[1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + \tilde{\theta}_i(\infty)\right]\right\}^3} \\
&\quad\times\left(\left[1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + \tilde{\theta}_i(\infty)\right]\left(1 + \frac{1+\varepsilon}{a_i}\right)\left[\tilde{\theta}_i(\infty) - 1\right] - \tilde{\theta}_i(\infty)\left\{1 + \tilde{\theta}_i(\infty)\left[1 + \frac{2(1+\varepsilon)}{a_i}\right]\right\}\right).
\end{aligned}$$

Thus if the following condition holds true:

$$\left[1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + \tilde{\theta}_i(\infty)\right]\left[\tilde{\theta}_i(\infty) - 1\right] \leq 0,$$

then the cross-partial derivative is negative for any $\tilde{\theta}_i(\infty) \in (0, 1]$. It is easy to verify that this condition holds for $\tilde{\theta}_i(\infty) \leq 1$ with any $\sigma_D \in (-1, 0)$, as desired in part (i).

Step 3. If, however, $\tilde{\theta}_i(\infty) > 1$, then we see that

$$\left[1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + \tilde{\theta}_i(\infty)\right]\left(1 + \frac{1+\varepsilon}{a_i}\right)\left[\tilde{\theta}_i(\infty) - 1\right] < \tilde{\theta}_i(\infty)\left\{1 + \tilde{\theta}_i(\infty)\left[1 + \frac{2(1+\varepsilon)}{a_i}\right]\right\}$$

is equivalent to

$$1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) \quad < \quad \underbrace{\frac{\tilde{\theta}_i(\infty)\left\{2 + \frac{1+\varepsilon}{a_i}\left[1 + \tilde{\theta}_i(\infty)\right]\right\}}{\left(1 + \frac{1+\varepsilon}{a_i}\right)\left[\tilde{\theta}_i(\infty) - 1\right]}}_{>0}.$$

Also, noting that

$$\frac{\tilde{\theta}_i(\infty)\left\{2 + \frac{1+\varepsilon}{a_i}\left[1 + \tilde{\theta}_i(\infty)\right]\right\}}{\left(1 + \frac{1+\varepsilon}{a_i}\right)\left[\tilde{\theta}_i(\infty) - 1\right]} - 1 = \frac{1 + \tilde{\theta}_i(\infty) + \frac{1+\varepsilon}{a_i}\left[1 + \tilde{\theta}_i(\infty)^2\right]}{\left(1 + \frac{1+\varepsilon}{a_i}\right)\left[\tilde{\theta}_i(\infty) - 1\right]} > 0,$$

the desired assertion in part (ii) follows. ■

Proof of Proposition 3.5. We shall complete the proof in 3 steps.

Step 1. If $\tilde{\theta}_i(\infty) < 1$, then for the two determinant terms of

$$\operatorname{sgn}\left\{\frac{\partial^2 T_i'(y_i(\infty))}{\partial\left(\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)\right)\partial(1/a_i)}\right\}$$

established in Step 2 of Proposition 3.4 we see the following facts: for the first term we have

$$1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + (1+\varepsilon)(1/a_i)\left[1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + \tilde{\theta}_i(\infty)\right] > 0$$

is equivalent to

$$\alpha_i(\infty) \quad < \quad \underbrace{\left(-\frac{\lambda_i}{\gamma_i}\right)\frac{1 + \frac{1+\varepsilon}{a_i}\left[1 + \tilde{\theta}_i(\infty)\right]}{1 + \frac{1+\varepsilon}{a_i}}}_{>0}$$

and also for the second term we have

$$\left[1 + \frac{\gamma_i}{\lambda_i}\alpha_i(\infty) + \tilde{\theta}_i(\infty)\right]\left(1 + \frac{1+\varepsilon}{a_i}\right)\left[\tilde{\theta}_i(\infty) - 1\right] > \tilde{\theta}_i(\infty)\left\{1 + \tilde{\theta}_i(\infty)\left[1 + \frac{2(1+\varepsilon)}{a_i}\right]\right\}$$

is equivalent to

$$\alpha_i(\infty) \quad > \quad \underbrace{\left(-\frac{\lambda_i}{\gamma_i}\right)\frac{1 + \tilde{\theta}_i(\infty) + \frac{1+\varepsilon}{a_i}\left[1 + \tilde{\theta}_i(\infty)^2\right]}{\left(1 + \frac{1+\varepsilon}{a_i}\right)\left[1 - \tilde{\theta}_i(\infty)\right]}}_{>0}.$$

Noting that

$$\frac{1 + \frac{1+\varepsilon}{a_i}\left[1 + \tilde{\theta}_i(\infty)\right]}{1 + \frac{1+\varepsilon}{a_i}} > \frac{1 + \tilde{\theta}_i(\infty) + \frac{1+\varepsilon}{a_i}\left[1 + \tilde{\theta}_i(\infty)^2\right]}{\left(1 + \frac{1+\varepsilon}{a_i}\right)\left[1 - \tilde{\theta}_i(\infty)\right]}$$

is equivalent to

$$0 > 2\tilde{\theta}_i(\infty)\left(1+\frac{1+\varepsilon}{a_i}\right)\left[1+\frac{1+\varepsilon}{a_i}\tilde{\theta}_i(\infty)\right],$$

which however is an immediate contradiction, so we must have

$$\frac{1+\frac{1+\varepsilon}{a_i}\left[1+\tilde{\theta}_i(\infty)\right]}{1+\frac{1+\varepsilon}{a_i}} < \frac{1+\tilde{\theta}_i(\infty)+\frac{1+\varepsilon}{a_i}\left[1+\tilde{\theta}_i(\infty)^2\right]}{\left(1+\frac{1+\varepsilon}{a_i}\right)\left[1-\tilde{\theta}_i(\infty)\right]},$$

by which and the facts that $\gamma_i/\lambda_i < 0$ for $\sigma_D \in (0,1)$ and $\partial(\gamma_i/\lambda_i)/\partial\sigma_D < 0$ the desired assertion in part (i) follows.

Step 2. If $\tilde{\theta}_i(\infty) = 1$, then we get from Step 2 of Proposition 3.4 that

$$\frac{\partial^2 T_i'(y_i(\infty))}{\partial\left(\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)\right)\partial(1/a_i)} = \frac{-2(1+\varepsilon)\left(1+\frac{1+\varepsilon}{a_i}\right)}{\left\{1+\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)+(1+\varepsilon)(1/a_i)\left[2+\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)\right]\right\}^3}.$$

Also, since we have that

$$1+\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)+(1+\varepsilon)(1/a_i)\left[2+\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)\right] > 0$$

is equivalent to

$$\alpha_i(\infty) \;<\; \underbrace{\left(-\frac{\lambda_i}{\gamma_i}\right)\frac{1+2\frac{1+\varepsilon}{a_i}}{1+\frac{1+\varepsilon}{a_i}}}_{>0},$$

the required assertion in part (ii) immediately follows.

Step 3. If $\tilde{\theta}_i(\infty) > 1$, then for the second term determining

$$\operatorname{sgn}\left\{\frac{\partial^2 T_i'(y_i(\infty))}{\partial\left(\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)\right)\partial(1/a_i)}\right\}$$

we have that

$$\left[1+\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)+\tilde{\theta}_i(\infty)\right]\left(1+\frac{1+\varepsilon}{a_i}\right)\left[\tilde{\theta}_i(\infty)-1\right] > \tilde{\theta}_i(\infty)\left\{1+\tilde{\theta}_i(\infty)\left[1+\frac{2(1+\varepsilon)}{a_i}\right]\right\}$$

is equivalent to

$$\alpha_i(\infty) \;<\; \underbrace{\left(-\frac{\lambda_i}{\gamma_i}\right)\frac{1+\tilde{\theta}_i(\infty)+\frac{1+\varepsilon}{a_i}\left[1+\tilde{\theta}_i(\infty)^2\right]}{\left(1+\frac{1+\varepsilon}{a_i}\right)\left[1-\tilde{\theta}_i(\infty)\right]}}_{<0},$$

which however is an immediate contradiction. As a result, we must have

$$\left[1+\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)+\tilde{\theta}_i(\infty)\right]\left(1+\frac{1+\varepsilon}{a_i}\right)\left[\tilde{\theta}_i(\infty)-1\right] < \tilde{\theta}_i(\infty)\left\{1+\tilde{\theta}_i(\infty)\left[1+\frac{2(1+\varepsilon)}{a_i}\right]\right\}.$$

Thus, using the first determinant term $1+\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)+(1+\varepsilon)(1/a_i)\left[1+\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)+\tilde{\theta}_i(\infty)\right]$, the required assertion in part (iii) follows. ■

Proof of Theorem 4.1. As usual, we derive the Stackelberg equilibrium by using backward induction. Thus, the Lagrangian of the follower country $-i$ is the same as in the case when these two countries play Nash, while the Lagrangian for the leader country i is different and reads as follows:

$$\begin{aligned}
&\mathcal{L}_i(\{U_i(w),l_i(w)\}_{w\in[\underline{w},\overline{w}]},\mu_i;\lambda_i,\gamma_i,\{\varsigma_i(w)\}_{w\in[\underline{w},\overline{w}]})\\
=\;& U_i(\underline{w})+\lambda_i\int_{\underline{w}}^{\overline{w}}\left\{[wl_i(w)-\varphi_i(U_i(w),l_i(w),\mu_i,\mu_{-i}(\mu_i))]\phi_i(U_i(w)-U_{-i}(w);w)-\frac{R}{\overline{w}-\underline{w}}\right\}dw\\
&+\varsigma_i(\underline{w})U_i(\underline{w})-\varsigma_i(\overline{w})U_i(\overline{w})+\int_{\underline{w}}^{\overline{w}}\left[\varsigma_i(w)h'(l_i(w))\frac{l_i(w)}{w}+\dot{\varsigma}_i(w)U_i(w)\right]dw\\
&+\gamma_i\left[\mu_i-\int_{\underline{w}}^{\overline{w}}\varphi_i(U_i(w),l_i(w),\mu_i,\mu_{-i}(\mu_i))f_i(w)dw\right].
\end{aligned}$$

Note that

$$\mu_{-i}=\int_{\underline{w}}^{\overline{w}}\varphi_{-i}(U_{-i}(w),l_{-i}(w),\mu_{-i},\mu_i)f_{-i}(w)dw,$$

making use of the Implicit Function Theorem produces:

$$\frac{\partial\mu_{-i}}{\partial\mu_i}=\frac{\int_{\underline{w}}^{\overline{w}}\frac{\partial\varphi_{-i}}{\partial\mu_i}f_{-i}(w)dw}{1-\int_{\underline{w}}^{\overline{w}}\frac{\partial\varphi_{-i}}{\partial\mu_{-i}}f_{-i}(w)dw}. \tag{37}$$

Assuming that there is no bunching of workers of different skills and the existence of an interior solution, then all of these first-order necessary conditions of Lagrangian $\mathcal{L}_i$ are the same as those in the proof of Theorem 3.1 but:

$$\begin{aligned}
\frac{\partial\mathcal{L}_i}{\partial\mu_i}=&-\lambda_i\int_{\underline{w}}^{\overline{w}}\left(\frac{\partial\varphi_i}{\partial\mu_i}+\frac{\partial\varphi_i}{\partial\mu_{-i}}\frac{\partial\mu_{-i}}{\partial\mu_i}\right)\tilde{f}_i(w)dw\\
&+\gamma_i\left[1-\int_{\underline{w}}^{\overline{w}}\left(\frac{\partial\varphi_i}{\partial\mu_i}+\frac{\partial\varphi_i}{\partial\mu_{-i}}\frac{\partial\mu_{-i}}{\partial\mu_i}\right)f_i(w)dw\right]=0,
\end{aligned}$$

where $\partial\mu_{-i}/\partial\mu_i$ is given by equation (37). The proof is thus complete. ■

Proof of Proposition 4.1. Applying condition (a) and (16) to equation (37) shows that

$$\frac{\partial\mu_{-i}}{\partial\mu_i}=\frac{-\sigma_F}{1+\sigma_D},$$

substituting which into the formula of γ_i/λ_i shown in Theorem 4.1 reveals that

$$\frac{\gamma_i}{\lambda_i}=\frac{\sigma_F^2-(1+\sigma_D)\sigma_D}{(1+\sigma_D)^2-\sigma_F^2}\tilde{F}_i(\overline{w}).$$

We thus obtain:

$$\frac{\partial(\gamma_i/\lambda_i)}{\partial\sigma_D} = -\frac{\sigma_F^2 + (1+\sigma_D)^2}{\left[(1+\sigma_D)^2 - \sigma_F^2\right]^2}\tilde{F}_i(\overline{w}) < 0$$

and

$$\frac{\partial(\gamma_i/\lambda_i)}{\partial\sigma_F^2} = \frac{1+\sigma_D}{\left[(1+\sigma_D)^2 - \sigma_F^2\right]^2}\tilde{F}_i(\overline{w}) > 0.$$

As a result, using chain rule, Corollary 4.1 and Proposition 3.3 gives rise to:

$$\frac{\partial^2 T_i'(y_i(\infty))}{\partial\sigma_D\partial(1/a_i)} = \underbrace{\frac{\partial^2 T_i'(y_i(\infty))}{\partial\left(\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)\right)\partial(1/a_i)}}_{<0} \cdot \underbrace{\alpha_i(\infty)\frac{\partial(\gamma_i/\lambda_i)}{\partial\sigma_D}}_{<0} > 0$$

and

$$\frac{\partial^2 T_i'(y_i(\infty))}{\partial\sigma_F^2\partial(1/a_i)} = \underbrace{\frac{\partial^2 T_i'(y_i(\infty))}{\partial\left(\frac{\gamma_i}{\lambda_i}\alpha_i(\infty)\right)\partial(1/a_i)}}_{<0} \cdot \underbrace{\alpha_i(\infty)\frac{\partial(\gamma_i/\lambda_i)}{\partial\sigma_F^2}}_{>0} < 0$$

for $\forall\sigma_D \in (-1,0)$ and $\tilde{\theta}_i(\infty) < 1$, as desired. For the other cases, we can similarly show that the predictions of Nash equilibrium carry over to the current Stackelberg equilibrium. ■

Proof of Proposition 4.2. It follows from Theorem 4.1 that

$$\frac{\partial(\gamma_i/\lambda_i)}{\partial(\partial\mu_{-i}/\partial\mu_i)} = \frac{\Lambda}{\left[1 - \int_{\underline{w}}^{\overline{w}}\left(\frac{\partial c_i(w)}{\partial\mu_i} + \frac{\partial c_i(w)}{\partial\mu_{-i}}\frac{\partial\mu_{-i}}{\partial\mu_i}\right)f_i(w)dw\right]^2}$$

where

$$\Lambda \equiv \left[1 - \int_{\underline{w}}^{\overline{w}}\frac{\partial c_i(w)}{\partial\mu_i}f_i(w)dw\right]\int_{\underline{w}}^{\overline{w}}\frac{\partial c_i(w)}{\partial\mu_{-i}}\tilde{f}_i(w)dw + \int_{\underline{w}}^{\overline{w}}\frac{\partial c_i(w)}{\partial\mu_{-i}}f_i(w)dw\int_{\underline{w}}^{\overline{w}}\frac{\partial c_i(w)}{\partial\mu_i}\tilde{f}_i(w)dw.$$

If Assumption 2.1 holds, then $\Lambda > 0$, and hence

$$\frac{\partial(\gamma_i/\lambda_i)}{\partial(\partial\mu_{-i}/\partial\mu_i)} > 0.$$

Also, using Theorem 4.1 and Assumption 2.1 again gives rise to $\partial\mu_{-i}/\partial\mu_i > 0$. Since we get from the optimal tax formula in Theorem 3.1 that optimal marginal tax rates strictly increase in γ_i/λ_i and $\partial\mu_{-i}/\partial\mu_i = 0$ in the Nash equilibrium, the required assertion accordingly follows. ■